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The Reality Problem in Quantum Mechanics

PAUL S. EPSTEIN
California Institute of Technology, Pasadena, California

1. The interchange of ideas between science and philosophy, which has been extensive at all times, has become in recent years even wider and more fruitful. In our own memory Einstein's *theory of relativity* was the outstanding instance of a profound influence of philosophic points of view on physical science. This example also shows that, in the friendly interchange, science is by no means always at the receiving end: if relativity was of immense importance for physics, it also settled age-old philosophic problems. However, at present, the part of physics which is the most fertile field of novel and unexpected conceptions undoubtedly is *quantum mechanics*. Heisenberg's principle of indeterminacy made us realize the inherent and unavoidable inaccuracy of mechanical description (in terms of positions and velocities) as well as the limitations of the classical principle of causality. It is the fountainhead from which the present flood of epistemological discussion springs. Indeed, in the last few years several books and many magazine articles on philosophy were written by physicists. When these philosophic writings are examined, it is found that they are mainly concerned with two topics. One is the bearing of modern scientific concepts on religious and ethical questions, the other is their relation to the problem of reality.

The second topic, *the problem of reality*, is as much a problem of physics as of philosophy, since it deals with the interpretation of the data of physical observation. If I select it as the subject of my paper, I am not so bold as to attempt its solution. I shall be content merely to review and criticize some of the views expressed by others in recent physical literature.

2. It will be appropriate to begin with a brief statement of the problem as it is usually formulated by philosophers. Any idea of the physical universe, which we may have formed, is derived from immediate sense data, *our sensations*, which form the raw material of our experience. To take a rather trite example: how do I know that a writing desk is before me? The answer is that I have a visual sensation of a polished surface; moreover, by stretching out my hand, I receive a tactile sensation; by bending over, I can obtain a visual sensation of the side and back surfaces. I know from experience that all these different sensory aspects go always together and form a single complex: when I experience one of them, it is within my power to evoke the others. So I come to interpret these sensations as different perceptions of one and the same thing, namely, the writing desk. The idea of a thing is thus obtained by the mental act of correlating a complex of sensations.

The *problem of reality* is the question of the status of things while they are not being perceived. The opinions of modern philosophers on this subject lie between two extreme positions, represented by two traditional schools of thought. The school of *dualism* regards the world of things as having a reality beyond the observing mind and *independent of it*. This view is closely akin to the so-called "naive realism" of the non-

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philosophic layman: the outer physical world and the observer are for him two separate domains. In the words of the poet (R. H. Stoddard):

> We have two lives about us,
> Two worlds in which to dwell,
> Within us and without us.

For the dualist, the correlation of the two domains is such that the perceptions and sensations of the observing mind are interpreted as impressions of really existing things of the independent outer world (called in philosophy "things-in-themselves") which happen to enter the field of consciousness. Of course, the things-in-themselves only produce these impressions and are not identical with them.

On the other hand, the *phenomenologists*, also called *sensationalists*, deny the separate existence of the perceived object and the perceiving subject. All sensations are, according to them, on the same level; the sensations of the so-called outer world and the inner perceptions of the subject (like phantasies) form one single stream of consciousness. Thus the distinction between a world of appearance (which is directly perceived) and a world of things-in-themselves (which is indirectly inferred) is regarded as illusory. The extreme form of this point of view is the so-called *solipsism*, which claims that all that we perceive and experience (in the words of E. A. Poe) ". . . is but a dream within a dream."

3. The two extreme points of view on the problem of reality, as well as the intermediate positions, have been amply discussed and criticized by philosophers. Though much of their criticism is profound and penetrating, it does not meet the direct needs of the physicist who is trying to apply the general considerations to special questions. When he attempts to analyze the situations in his own way, he finds upon closer examination that it is helpful to divide the problem of reality into at least three independent parts. These three components must be disentangled before it is possible to state the point of view of the physicist.

The first component problem which presents itself may be called the *psychological problem*. It arises from the simple fact that our subjective sense impressions of nature have a counterpart in objective observations with instruments. For instance, to the subjective sensations of "hot" and "cold" correspond objective readings of the thermometer; the sense impression of red or blue can be objectively described and measured in terms of wave-lengths of light. Thus, we have, on one hand, a world of subjective sensations, on the other, a world of controlled observations reduced to numerical measurements. This aspect of the problem is so elementary that it is of little interest to science. Indeed, every branch of science develops its own criterions of objectivity and gradually eliminates all subjective elements of description, replacing them by more and more objective ones. We shall take it for granted that in physics this process has progressed so far that the world of physics with which we have to deal is a world of reproducible physical observations reduced to numerical data.

As to the bearing of this situation on philosophy, it must be pointed out that the two worlds just mentioned are both worlds of observations. Only, in the one world, they are crude observations colored by our sense impressions; in the other, they are refined and made objective. Hence, the question of the existence of an independent outer domain of the things-in-themselves does not at all enter into the psychological problem. It is true that the possibility of making our observations reasonably objective and reproducible strongly supports the conclusion that a point of view regarding them as produced by things independent of the observer is, at least, legitimate. However, a positive assertion to this effect would be metaphysical because it is not capable of proof. Indeed, the extreme solipsistic point of view that all "is but a dream within a dream," though entirely sterile, is always possible and free of contradictions. On the other hand, we remain within physics if we turn the problem round and ask to what extent the extreme dualistic view is tenable. *Are the accumulated observational data which we possess of such a nature that they can be interpreted as the perceptions of an outer world completely independent of the observer*, or are there elements in these observations that make such an interpretation inadequate? We shall state the other two components contained in the problem of reality in a form appropriate for answering this question.

4. The second component which we have to discuss may be called the *philosophic problem of reality*. To make this problem clear, it will be best to use the historical approach and to explain what form it took in Kant's philosophy. Kant held the dualistic point of view and believed that our observations are impressions produced by an outer world of *things-in-themselves*. Of great interest to him were the qualities of ex-

tension and duration which form part of our perceptions. He put himself the question: do these qualities really belong to the outer world, or are they merely superimposed onto it by our mind? Are they inherent in the things-in-themselves or are they the form of consciousness under which we perceive them? Kant answered this question in the second sense: space and time are, according to him, constructions of the human mind; they form, so to speak, the screen onto which the mind projects its sensations of the outer world. The mind cannot conceive of the world in any other way than in the space frame provided by Euclidean geometry. The axioms of Euclidean geometry were considered by him as inescapable necessities of thinking. A similar role was attributed to the principle of causality, which he also did not consider as inherent in the *things-in-themselves* but as another necessity of thinking superimposed upon the outer world by the human mind.

The scientific discoveries of the last 150 years have disproved Kant's contentions. We know at present that the Euclidean axioms are not necessities of thinking. The human mind can conceive of an infinite number of geometries, while the geometry appropriate to the description of the physical world is determined by the properties of this world as revealed by observations. Similarly, the principle of causality is neither a necessity of thinking nor always appropriate to the description of nature.

These considerations about the physical space-time and about causality received their final solutions only in the last decades. They are typical of what we have called the *philosophic problem of reality*. In its conception this problem is quite different from the first component, or psychological problem of reality. In fact, in that case we found that our sensations and feelings falsify our impressions of nature and that these influences can be removed by a proper technic of objective observations. On the contrary, in the cases relating to the philosophic problem the claim is that limitations of our capabilities for logical thinking press objective observations into a mold which is not predetermined by the nature of these observations. Of course, such effects, rooted in our mental organization, cannot be removed.

In practice, however, it is hard to distinguish a philosophic problem of reality from a psychological. It is certain that the examples discussed by Kant (namely, space-time and causality) have turned out to be not necessities of thought but merely deeply ingrained habits of thinking. Nevertheless, several contemporary authors hold that some features of the world of observations are due to our mental organization.

5. The main champion of the point of view that our scientific world picture is greatly influenced by our mental equipment was the late Sir Arthur Eddington.[1] In fact, he went so far as to question whether science is really discovering a physical world or creating one. Of several arguments which he adduces to the support of his contention, two are worthy of a closer examination while the rest seem less convincing.

The first of these arguments refers to the peculiarities of *indistinguishable particles*. It is recognized by modern physics that a group of particles which are all identical (for instance, all electrons) is not subject to the same statistical laws as is a group of particles which are all different. Eddington saw in this the influence upon the laws of nature of our mental make-up, namely, of our inability to distinguish among the particles.[2] This situation is quite different in its nature from the limitations imposed by the principle of indeterminacy, which is the statement of our inability to measure simultaneously *position and momentum*. In fact, the reason for our inability in this respect is the unpredictable disturbance produced by the measuring instruments. It is the contention of quantum mechanics that this is a *law of nature* which is quite independent of the functioning of our mind. On the contrary, the statistics of identical particles was considered by Eddington as due to our mental limitations.

A few years ago I happened to express on this subject a view quite different from Sir Arthur's, namely, that the peculiar properties of indistinguishable particles have their root precisely in the principle of indeterminacy and have nothing to do with our mental comfort. As my argument was published in a place where few readers are likely to look for it,[3] I shall reproduce and elaborate it here.

Let us consider two moving, identical atoms, first, from the point of view of classical mechanics. According to this point of view, positions and momentums are, on principle, measurable

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[1] A. Eddington, *The philosophy of physical science* (1939).
[2] Reference 1, pp. 32–33.
[3] P. S. Epstein, *Textbook of thermodynamics* (Wiley, 1937), footnote on p. 260.

with unlimited accuracy; that is, any present failure to achieve such an accuracy is due to difficulties of insufficient technic which will be surmounted by super-observers of the future. Hence we can imagine that each of the two moving atoms is in charge of a super-observer who continuously watches its position. Each super-observer knows at any moment where his atom is, so that there is no danger of confusing it with the other atom. Although the atoms have identical properties, yet it is always known which is which, as they can be told apart by the continuity of their positions. Thus the statistics appropriate for them is classical statistics, which assumes that they *can be told apart.*

Let us now regard the same situation from the point of view of quantum mechanics. Even a super-observer cannot accurately measure the positions at close time intervals. Indeed, if he could do so, he would be also able accurately to infer the velocity and momentum, in contradiction with the principle of indeterminacy. As the two atoms approach each other, there comes a moment when their mutual distance is of the same order of magnitude as the indeterminacy of their positions. Then they are indistinguishable in the sense that the super-observers (who identified them by the continuity of their positions) can no longer say which is which.

Let us follow this argument through quantitatively for a gas of temperature T consisting of identical atoms. We have to start from the exact formulation of the principle of indeterminacy,

$$\langle(\Delta p_x)^2\rangle_{Av}\cdot\langle(\Delta x)^2\rangle_{Av}=\tfrac{1}{4}\hbar^2, \tag{1}$$

where $\langle(\Delta p_x)^2\rangle_{Av}$ is the mean square of the error (or indeterminacy) Δp_x with which the component p_x of the momentum is known, $\langle(\Delta x)^2\rangle_{Av}$ is the mean square of the error in the coordinate x of position, and $\hbar$ is a constant, known as the *quantum of action.*

In our particular example something is known about the momentums of the individual atoms because they belong to a gas of temperature T: according to the principles of classical statistics, the mean square of the component p_x is determined by the temperature according to the equation $\langle p_x^2\rangle_{Av}=\frac{1}{2}kT/m$, where k denotes Boltzmann's constant and m the mass of the atom. This mean square refers as well to the momentums of all atoms of the gas as to successive measurements of the momentum of a single atom.

It is obvious, however, that the mean square of the indeterminacy of any quantity cannot be larger than the mean square of the quantity itself. Hence, $\langle(\Delta p_x)^2\rangle_{Av}\leqslant\langle p_x^2\rangle_{Av}$, or

$$\langle(\Delta p_x)^2\rangle_{Av}\leqslant\tfrac{1}{2}kT/m. \tag{2}$$

Combining this result with Eq. (1), we obtain for the indeterminacy of position of an atom in the gas,

$$\langle(\Delta x)^2\rangle_{Av}\geqslant\tfrac{1}{2}\hbar^2m/kT, \tag{3}$$

which holds as long as classical statistics is a sufficient approximation. It is obvious that the atoms must be indistinguishable, as to their positions, if this quantity $\langle(\Delta x)^2\rangle_{Av}$ is materially larger than the square of the mean distance l between neighboring molecules. If v denotes the molal volume of the gas and n_A the Avogadro number, the number of molecules occupying 1 cm³ is n_A/v and the mean distance, in the x-direction, between neighboring molecules is $l=(v/n_A)^{1/3}$. The condition $\langle(\Delta x)^2\rangle_{Av}\gg l^2$ will certainly be satisfied when $\frac{1}{2}\hbar^2m/kT\gg(v/n_A)^{2/3}$, or when

$$\Theta\equiv 2kTv^{2/3}/\hbar^2mn_A^{2/3}\ll 1. \tag{4}$$

Under this condition classical statistics will not apply. On the contrary, the atoms can be well told apart, and classical statistics is appropriate, when $\langle(\Delta x)^2\rangle_{Av}$ is very much smaller than l^2, or when $\Theta\gg 1$.

Thus, we can expect either negligible or large deviations from classical statistics according to whether the number Θ, defined by formula (4), is large or small compared with unity. This is, in fact, nearly the same criterion which we find in the detailed statistical theory of gases.

6. The other remark of Eddington's which we wish to discuss here is to the following effect. While we may have learned to make our observations objective, a subjective element is brought in by the fact that we have actually made only a limited number of selected observations. Hence, Eddington designated the picture which we possess of the universe as *selective subjectivism.*

The answer which may be given to this remark is as follows. While it is true that we have not yet tried all possible experiments, it is the intention and program of science to extend its inquiry to all observations that can be performed. If Eddington's remark has any meaning, it can only amount to the contention that the limits of our knowledge of the world are set, not by the physical nature of this world (which includes our instruments), but by our mental disabilities, because it is beyond the power of our imagination to devise certain observations. Such a point of

view is not logically impossible, but it falls into the class of metaphysical speculations which we have excluded. In fact, by the very definition of observations of which we cannot conceive, their existence can never be proved or disproved.

Returning to the general problem, it is safe to make the following statement. If any law governing the world of observation were known to us *a priori*, before any observations were made, this would be an indication that the properties of this world are partly due to our mental organization. Several authors in reviewing Eddington's work attribute to him the contention that *a priori* laws in the foregoing sense exist. A few passages from Eddington's book, where he contrasts the certainty of what he calls *a priori* laws with the limited validity of laws found *a posteriori*, seem to substantiate this accusation. However, Eddington himself was not entirely sure[4] that he was using the term "*a priori* knowledge" in its recognized meaning; and a closer examination of his text proves beyond doubt that he was not. Indeed, the recognized meaning which philosophers attribute to this term, and in which we have used it above, is that of an "inescapable necessity of thinking." Since Eddington mentions the principle of relativity as an example of an *a priori* law, he certainly meant no such thing. Having himself contributed to the development of the theory of relativity, he was well aware that it is not a necessity of thought, as evidenced by the fact that 19th century physics was nonrelativistic and that relativity was accepted in the 20th century only with great reluctance.

Eddington used the term "*a priori* knowledge" in a sense which would be better described by saying "knowledge derived from epistomological investigation." Now, the usual pattern of such an investigation is as follows. In a previously existing but inadequate theory a hidden assumption is discovered and shown to be arbitrary. As a rule, the number of other possible assumptions, by which the arbitrary one can be replaced, is limited. Hence, very few observational data—and these of a fairly general nature—are needed to find the correct theory. A case in point is the restricted theory of relativity: the crucial stage in its development was Einstein's realization that the physical space-and-time measurements were arbitrarily assumed to have the properties of the absolute space-time. Once this was understood, very little knowledge of physical facts was necessary to find the true properties of the physical space-time.

Understood in their true sense, Eddington's ideas of the superior security of epistemologically discovered laws, far from being revolutionary, are contentions which any physicist will gladly grant. The contrary impression of some critics who "consider these ideas to be a considerable danger to the sound development of science"[5] is based on a misunderstanding provoked by Eddington's unconventional use of philosophic terms.

7. We can now turn to the third and last component contained in the problem of reality, which we shall call the *physical problem of reality*. The world with which the physicist has primarily to deal is the world of physical measurements, which we have to some extent already described. For brevity, we shall designate it as *the world of the observer*. As has already been mentioned, the scientist not only obtains perceptions *directly* through his sense organs, but he greatly enhances and refines the powers of these organs by using them in conjunction with scientific instruments. These mechanical tools may be regarded as artificial extensions of the person of the observer and, for many purposes, can be treated as parts of him. As we have agreed that subjective elements of observation should be excluded, it makes no difference whether an object, say a star, was observed directly or photographed on a plate which was later examined by the observer. As far as the physicist is concerned, the human eye is an optical instrument—as it were, a built-in camera—and the other sense organs have a similar status. It is immaterial whether the observation is made directly by the observer, obtained in his absence by a self-recording instrument, or carried out by another person and passed on to him with a full description. The world of the observer is simply the *totality of all critically sifted observations, no matter how obtained*.

It is important to note that the world of the observer is a *physical world* inasmuch as the objective observations, of which it consists, are subject to physical laws. If the observer is a dualist, he opposes to it another *inferred* physical world, interpreting his observations as the manifestations of a nature that exists beyond him and his instruments, and that will continue to exist even if all observers should perish. The *physical*

[4] Reference 1, p. 24.

[5] M. Born, *Experiment and theory in physics* (Cambridge, 1943), p. 4.

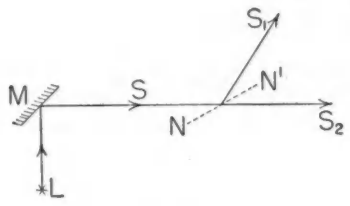

FIG. 1.

dualism consists in the opposition of these two physical worlds: the one directly observed, the other inferred.

We may now put the question whether, on this level, the *dualistic* point of view is necessary or may be replaced by the *phenomenological*, which recognizes only the world of the observer and avoids mentioning any reality beyond it. We stated previously that the phenomenological solution is always possible, being free of inner contradictions. However, this is a philosophic approach to a question which is not a question of philosophy. Indeed, both worlds under discussion (the observed world and the hypothetically inferred) being subject to physical laws, the problem before us is a problem of physics. For deciding between the two points of view, it is not sufficient that either of them be self-consistent; the question is: which is better suited to the logical description of the accumulated scientific observations? If this criterion be used, the problem is not necessarily indeterminate. In fact, the mathematical formalism of quantum dynamics—which has been developed to account for the observational data—is plainly based on a modified dualism. One part of this formalism consists of the partial differential equations to which nature is subject, as long as it is unobserved; another part refers to the erratic disturbances which are introduced through the unavoidable interference of the observer with nature, in the pursuit of his business of securing measurements. Thus, the interruption of causality, characteristic of the quantum theory, takes place at the points of interaction between the outer world and the instrumental paraphernalia of the observer. We called this dualism *modified* because it is unlike the naive realism in that the world of the observer is not a replica of the unobserved world but is different in view of the modifications introduced by the process of measurement.

The incomplete correspondence between the observed and the unobserved worlds involves the consequence that the acceptance of the dualistic point of view does not yet resolve the problem of reality but only sublimates it onto a different plane. While it is admitted that the world continues to exist while unobserved, there remains the question about the exact properties of this unobserved world. Let us illustrate this by an example. Certain complexes of observations (which always occur together) are called electrons, other complexes are called protons, and so forth. It is not questioned that, under most conditions, electrons or protons exist also between observations; the question is only about the reality of their space-time description. The positions and momentums of electrons and protons measured during observations are usually called their *observables*. The principle of indeterminacy, mentioned above, imposes restrictions on these observables in that position and momentum cannot be simultaneously accurately measured. Hence, if one of the two—either position or momentum—is observed accurately, the other remains unobserved. These unobserved quantities, as well as the positions and momentums which may be attributed to electrons and protons between observations, are sometimes referred to as *unobservables*. Hence, the problem of reality boils down to the question whether, or to what extent, reality can be assigned to the unobservables; meaning by this, whether it is possible to make about them significant statements. Consequently, the physical problem of reality, so circumscribed, is a question of physics pure and simple. There is no doubt that those of its points which are at present still controversial will be eventually settled by physical methods.

8. We pass now from the abstract to the concrete, namely, to the discussion of some specific examples of unobservables that have no reality, according to the standards of the quantum theory. We have already mentioned one example, which is an immediate consequence of the principle of indeterminacy: *when the momentum of a particle is accurately known, its coordinate of position has no physical reality*. Indeed, under these conditions the coordinate cannot be measured, and there is no experimental way of finding out what it is.

Another case in point is a simple but very instructive example described in most textbooks of quantum dynamics.[6] It will be well to treat it here at some length. Let us consider a light wave

[6] For example, P. A. M. Dirac, *The principles of quantum mechanics* (Ed. 2, 1935), p. 4.

S (Fig. 1) starting from the source L and falling onto the half-silvered glass plate NN'. It is thus divided into two waves S_1 and S_2 which are *coherent*, meaning by this that they will exhibit interference when recombined. So far we have described the situation from the point of view of the wave theory; what is the picture from the point of view of photon propagation? Let us suppose that the intensity is so low that the photons follow one another at large distances, so that we have to deal only with one photon at a time. We can insert into the light path a movable mirror M, whose recoil under the impact of a photon will not destroy our light wave but will serve as an indicator of the presence in the apparatus of one and only one photon.

How does the interference come about under these conditions? It is the contention of quantum mechanical theory that every photon interferes only with itself, because it does not move either exclusively in the beam S_1 or exclusively in the beam S_2, but in both beams simultaneously. If we designate by ψ_1 the state of our system in which the photon belongs to the beam S_1 and by ψ_2 the state in which it belongs to the beam S_2, then quantum mechanics recognizes the composite state

$$\psi = c_1\psi_1 + c_2\psi_2, \qquad |c_1|^2 + |c_2|^2 = 1, \tag{5}$$

in which the photon belongs *to both beams simultaneously*. The symbols c_1, c_2 are two constants such that the squares of their absolute values $|c_1|^2$, $|c_2|^2$ represent the fractional weights with which the beams S_1, S_2 share in the photon. In our particular case, when the plate is half-silvered, the intensity of the two beams must be the same. From the point of view of the photon description, this means that

$$|c_1|^2 = |c_2|^2 = 1/2. \tag{6}$$

Let us now modify the conditions by putting into the path of each beam, S_1 and S_2 a movable mirror, M_1 and M_2, respectively (Fig. 2). Again we arrange it so that these mirrors can recoil under the impact of the photon *without destroying the wave*. If the mirror M_1 recoils, it is certain that the photon is in the beam S_1. The mathematical description of the state is, therefore, no longer given by our previous Eqs. (5), (6) but by $\psi = \psi_1$ or, in other words, by the Eqs. (5) with the weight factors,

$$|c_1| = 1, \quad c_2 = 0. \tag{7}$$

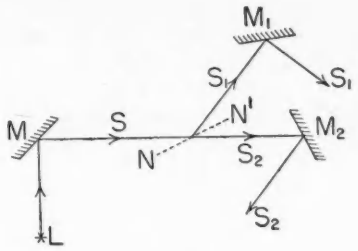

FIG. 2.

As the photon is now entirely in the beam S_1 and the second beam S_2 is completely devoid of it, the photon can no longer interfere with itself: the two beams have become *incoherent*.

Thus, the observation of the photon by means of the mirror M_1 has as its consequence a sudden change of state of the system. At the moment when the mirror recoils the following changes take place: from the point of view of the *photon description* the state represented by Eqs. (5) and (6) abruptly goes over into that represented by Eqs. (5) and (7); from the point of view of the *wave description*, the two waves S_1 and S_2 suddenly become incoherent. This sudden change of properties is brought nearer to our understanding by the fact that the moving mirror really produces a disturbance of wave motion and supplies a sufficient reason for the transition of state.

It goes without saying that similar considerations apply also to the other mirror: if, instead of M_1, the mirror M_2 recoils, it becomes certain that the photon is in the beam S_2; at this moment, the state changes from (5), (6) to (5) with the weights

$$c_1 = 0, \quad |c_2| = 1, \tag{8}$$

and the two beams become incoherent.

In fact, if both mirrors M_1 and M_2 are movable, one of them will necessarily recoil. Either M_1 will move, and the system will assume the state (5), (7); or M_2 will be displaced, and the system will jump into the state (5), (8). We have here a *disjunctive alternative* in the sense that one of the two possibilities will occur with necessity.

9. We have now described the properties of our system sufficiently to fit it into our general considerations. We have seen that its state is described by Eqs. (5), (6) in two cases: first, when no movable mirrors are interposed in the beams S_1, S_2 as in Fig. 1; second, when mirrors are interposed (Fig. 2) but the time is prior to the

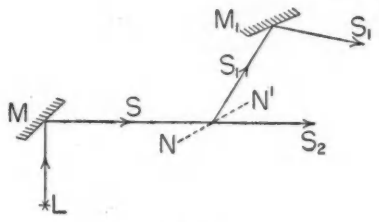

FIG. 3.

recoil of either the mirror M_1 or M_2. Under these conditions the location of the photon in the beam S_1 or the beam S_2 is an *unobservable*, as there is no way of deciding in which of the two it is. In fact, if we observe its position, say in the beam S_1, by means of the recoil of the mirror M_1, we know that it is in the beam *after the recoil*. Where it was before the recoil remains undecided, since the state of the system changes discontinuously at the moment of observation. In this sense, the location of the photon in one beam or the other *has no reality* in the two cases just mentioned. This is the reason why the quantum dynamical theory attributes it to both beams simultaneously.

The most interesting discussion referring to the problem of the reality of unobservables was started a few years ago in a joint paper by Einstein, Podolsky, and Rosen.[7] Considering a special class of systems, these authors found difficulties in the quantum mechanical description of them. Although their systems were of an entirely different mathematical structure from our simple example of the half-silvered plate, it is possible to use this example in explaining the nature of the difficulties pointed out by them. In fact, the example of the half-silvered plate has even certain advantages. Just because of its very simplicity, both the case of Einstein, Podolsky, and Rosen can be stated with its help in a more convincing way, and the answer of quantum mechanics to it can be made more plausible.

In order to understand the argument of Einstein, Podolsky, and Rosen, let us consider an arrangement which differs from that of Fig. 2 only in that the mirror M_2 is omitted (Fig. 3). The remarkable thing about it is that it gives us just as much information about the photon as the arrangement with the second mirror. Indeed, if the mirror M_1 recoils, we are certain that the photon is in the beam S_1 and that the state has suddenly jumped from (5), (6) to $\psi=\psi_1$. On the other hand, if the mirror M_1 does not move within a reasonable time after M has recoiled, we conclude, with equal certainty, that the photon is not in the beam S_1 and, hence, the coefficient c_1 is equal to naught. It follows then from the relation $|c_1|^2+|c_2|^2=1$ that $c_2=1$, or that the photon is in the beam S_2.

Therefore, the quantum dynamical interpretation of the second case (when the mirror did not move) must be quite analagous to that of the first case, namely, a sudden transition to a new state with a loss of coherence between the two beams. However, it must be noted that, in the absence of motion of the mirror, the geometric arrangement does not undergo any change. Hence, the explanation of the loss of coherence as due to a disturbance of the wave path cannot be invoked.

We stand, thus, before the same dilemma at which Einstein, Podolsky, and Rosen arrived:

(1) Either we must assume that the system undergoes a sudden change of state without being induced to this by any physical change in its conditions;

(2) Or, if this is not the case, the reason why the photon goes into the beam S_2 must be contained in the description of its initial state. Hence, Eqs. (5), (6) which deny reality to the location of the electron in either beam must be inadequate.

The above-mentioned authors were reluctant to admit the first horn of this dilemma—which, apparently, contradicts the law of sufficient reason—and concluded that the description provided by the theory of quantum mechanics must be incomplete. On the contrary, Niels Bohr,[8] answering their paper, holds the opposite view. Applied to our example, his statement is to the effect that while the mirror admittedly does not recoil, the fact *that it is able to recoil* in itself introduces a new element which is not taken into account in the foregoing analysis.

10. On the whole, the discussion just described did not stir up much excitement among physicists, doubtless because of the somewhat abstract character of the contentions involved in it. Indeed, the fundamental fact from which the difficulties flow is as follows: the lack of response of the movable mirror gives us the *mental certainty* that the photon has passed through the other beam. This can be also stated in a different way: we are certain that, if we had placed a movable mirror in the other beam, this second mirror would have recoiled.

[7] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).

[8] N. Bohr, Phys. Rev. **48**, 696 (1935).

Now, most physicists are not greatly disturbed by things that "would have happened, if . . . ." They feel that such questions smack of metaphysics and had better not be asked. Here is where our example of the half-silvered mirror has a decided advantage. It can be brought into a form in which the fundamental facts are no longer abstract so that the physicists have no longer the right to ignore them because they can be presented as the antithesis of two different physical observations.

To show this, let us build up our arrangement into an interferometer by adding to the first half-silvered plate NN' and the two mirrors M_1, M_2 a second half-silvered plate OO' (Fig. 4). We may omit the first mirror M of the preceding figures and instead make provision for observing the photon when it comes out from the half-silvered plate OO' in one of the beams S_3 or S_4.

(1) If *both mirrors*, M_1 and M_2, *are fixed*, the two branches S_1 and S_2 are coherent. All parts can be adjusted at such positions and angles that the whole intensity will pass into the beam S_3 and nothing will go out in the direction of the beam S_4. Hence, every photon which comes out of the interferometer will be observed in the beam S_3.

(2) We pass now to a construction of our apparatus in which *both of the mirrors* M_1, M_2 *are movable*. Then we know that one of them will recoil under the impact of the photon. At the moment of the recoil the coherence of the two branches is lost, so that there is no interference at the half-silvered plate OO'. As a result, each photon that reaches it has an equal chance of passing into the beam S_3 or into the beam S_4. The emerging photons are thus observed evenly distributed between the two beams.

Finally, we consider the case where the mirror M_2 is fixed and M_1 movable. If the movable mirror recoils, we certainly have, at the moment of the recoil, a change of state with loss of coherence, and we know that the photon has passed through the branch S_1. On the other hand, if the mirror M_1 does not recoil, we conclude with equal certainty that the photon coming out of the interferometer has passed through the branch S_2.

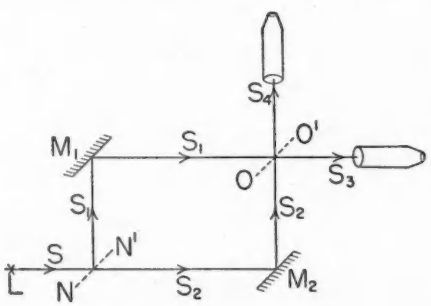

FIG. 4.

Thus, we arrive again at the dilemma of Einstein, Podolsky, and Rosen; but its horns can now be described in terms of two different observational results. Consider first the second horn: the assumption favored by these authors that there is *no change of state without a recoil*. If the description provided by the quantum theory is complete, this implies that the state remains the same as in the case when both mirrors are fixed, and that it has the same observational manifestations. In other words, the waves in the branches S_1 and S_2 are coherent, and all photons pass into the beam S_3. This is in flagrant contradiction with the conclusion drawn above that, in the absence of a recoil, the photon must have passed through the branch S_2. Indeed, we not only have to answer the question, put by Einstein, Podolsky, and Rosen, of why the quantum dynamical description of the state denies the reality of the location of the photon in the branch S_2, but we must face a new difficulty. If it is certain that the photon passed through the branch S_2, how did its interference come about? What did it interfere with, if the other branch was entirely free of photons? If the foregoing assumption is correct, the quantum mechanical description must not only be incomplete but radically wrong.

On the other hand, the difficulties which Einstein, Podolsky, and Rosen see in the first horn of the dilemma can be also brought nearer to the physicist by stating them in more concrete terms. Niels Bohr and the orthodox quantum dynamical theory favor the opposite assumption that, *although the mirror fails to recoil, a sudden transition of state*, with loss of coherence, *takes place*. We can, therefore, contrast the following two cases: first, both mirrors are fixed; second, one mirror is free to move but does not make use of this freedom. In both cases the geometric configurations of the wave paths are exactly the same. Yet, in the first case, all photons pass into the beam S_3, in the second case, they are evenly divided between S_3 and S_4.

11. What we have accomplished in working through our simple example is to impart a very acute form to the difficulties of the explanations given by the quantum theory. Fortunately, the

same example will help us to some extent to understand the resolution of these difficulties, granting in advance that we are quite unable to give a detailed description of what is actually happening and that any picture we make in terms of classical conceptions must necessarily be crudely simplified and distorted.

Let us go back to the interferometer, as illustrated by Fig. 4, with *two fixed mirrors* M_1 and M_2. The two states ψ_1 and ψ_2, corresponding to the location of the photon in the beams S_1 and S_2, respectively, are completely equivalent, so that the photon has in both branches the same energy. A system which possesses two different states ψ_1, ψ_2 of the same energy is called *degenerate*. If the energy of the system is known, it can still be either in the state ψ_1 or in ψ_2, or partially in ψ_1, partially in ψ_2, as expressed by the mathematical representation $\psi = c_1\psi_1 + c_2\psi_2$.

Suppose now that a degenerate system of any kind is subjected to a slight perturbation, meaning by this that the conditions are slightly changed. This will result in a small change of the energies of all its possible states. In particular, the energies of the states ψ_1 and ψ_2, previously equal, may change by different amounts. In this case, the two states are no longer equivalent in the perturbed condition, and the degeneration no longer exists. If it is known that the system is in only one energy state, it must be either in ψ_1 or in ψ_2, but it cannot be in both ψ_1 and ψ_2 simultaneously.

In this sense, the photon in the interferometer with fixed mirrors is a degenerate system. However, making one mirror movable amounts to a perturbation of the system which resolves the degeneracy. In fact, in passing through the branch with the movable mirror the photon has a different energy from that in the other branch, as it changes its energy in the collision. Hence, the states ψ_1 and ψ_2 are no longer energetically equivalent. The energy being an essential element of its description, the photon can have only one energy at a time. It is true that this energy is affected by a slight indeterminacy caused by the finite length of the wave train which is associated with the photon. However, we can safely assume that this indeterminacy is small compared with the energy loss in the collision with the recoiling mirror. Hence, in the case of a moving mirror, the photon cannot be in the states ψ_1 and ψ_2 simultaneously as they are states of different energies. It can only be either in the branch S_1 or in the branch S_2.

Of course, we cannot admit into our description any elements of action at a distance. Allowing for the unavoidable crudities of any classical picture, we must describe the process something like this. When the photon is at the first half-silvered plate NN', there is no criterion to tell that the two branches are nonequivalent. Therefore, the photon starts out as a degenerate double wave belonging to both branches and described by the formula $\psi = c_1\psi_1 + c_2\psi_2$. However, as the wave draws nearer and nearer to the mirrors, the lack of equivalence due to the mobility of one of the mirrors makes itself felt more and more. There comes a moment when this description is no longer adequate and the degeneracy must be resolved. In other words, the state must change into one of the two nondegenerate states $\psi = \psi_1$ or $\psi = \psi_2$.

If the state undergoes the sudden change into ψ_2, so that the movable mirror does not recoil, it cannot be said that this transition happened *without sufficient reason*. The reason lies in the fact that the wave symbolizing the photon was drawing near the sphere of action of the movable mirror.

We think this is a good illustration of the point of view advanced by Bohr,[8] in his above-mentioned paper, which is not easy to understand without working through a similar example: the point of view that the very *possibility* of a measurement may be, in itself, a sufficient reason for a sudden change of state, even if a physical disturbance of the system did not take place.

Summarizing, we come to the conclusion that the critical work of Einstein, Podolsky, and Rosen was highly important in contributing to the clarification of the conceptions of quantum dynamics but was in no way fatal to these conceptions. In particular, it did not prove that the unobservables have more reality than the quantum theory attributes to them.

Once you tamper with your basic concepts, philosophy is merely the marshaling of one main source of evidence, and cannot be neglected.—A. N. WHITEHEAD.

Rotations of a Rigid Body About a Fixed Point

E. L. Hill
University of Minnesota, Minneapolis, Minnesota

THE study of the rotations of a rigid body about a fixed point is of importance in many parts of physical and mathematical theory. Owing to this interest, numerous mathematical procedures and geometric constructions have been developed for the representation of finite rotations. Apart from the usual method of using the transformation equations for the Cartesian coordinates of a point, which is one of the most cumbersome of all,[1] we have Euler's theorem and its vector equivalent,[2] the quaternion method of Hamilton,[3] the stereographic projection of the sphere and the associated use of the Cayley-Klein parameters,[4] and the method of Clifford's numbers,[5] which has been rediscovered and used with particular effect in quantum mechanics[6] under the name of "spin variables," or *spinors*.[7] Naturally, these methods all give algebraically equivalent results, but they differ widely in the ease with which the calculations may be carried out. Of all of them, the method of spinors is particularly simple algebraically and deserves to be more widely known and used, even in elementary discussions.

In this paper the spinor method will be used for developing the basic formulas for the discussion of finite rotations, and these formulas will then be applied to some of the problems that arise in classroom discussion.

[1] This direct use of the Cartesian coordinates brings one to the method of differential operators developed by Lie in his theory of continuous groups. A general discussion is given in S. Lie and G. Scheffers, *Continuierliche Gruppen* (Leipzig, 1893).

[2] C. J. Coe, *Theoretical mechanics* (Macmillan, 1938), chap. 5; J. Ames and F. Murnaghan, *Theoretical mechanics* (Ginn, 1929), chap. 2; H. Lamb, *Higher mechanics* (Cambridge, 1929), chap. 1.

[3] W. R. Hamilton, *Elements of quaternions* (Longmans, 1901).

[4] E. T. Whittaker, *Analytical dynamics* (Cambridge, 1927), chap. 1; F. Klein and A. Sommerfeld, *Theorie des Kreisels* (Leipzig, 1897), vol. 1.

[5] W. K. Clifford, Am. J. Math. **1**, 350 (1878); G. Juvet and A. Schidlof, Bull. de la Société Neuchateloise des Sciences Naturelles **57**, 127 (1932).

[6] P. A. M. Dirac, Proc. Roy. Soc. **A117**, 610 (1928); **A118**, 351, (1928).

[7] The name *spinor* arises from the connection with the problem of the spin of the electron. For further details, reference may be made to books on quantum mechanics or to the review article by E. L. Hill and R. Landshoff, Rev. Mod. Phys. **10**, 87 (1938).

1. The Representation of Rotations by Spinors

Let σ_1, σ_2, σ_3 be three quantities that obey the algebraic rules

$$\sigma_1^2=1, \tag{1}$$

$$\sigma_1\sigma_2=-\sigma_2\sigma_1=i\sigma_3, \tag{1'}$$

together with the similar relations which are obtained from these by cyclic permutation of the indices (1, 2, 3). We combine these quantities into a symbolic "spin vector,"

$$\boldsymbol{\sigma}=\sigma_1\mathbf{i}+\sigma_2\mathbf{j}+\sigma_3\mathbf{k},$$

where $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ represent a set of orthogonal right-handed unit vectors with origin at the fixed point of the rotation. It is to be explicitly noted that the i appearing in Eq. (1') is $(-1)^{\frac{1}{2}}$ and is not to be confused with the unit vector $\mathbf{i}$, which will always appear here in bold-faced type.

We shall be concerned only with direct algebraic manipulations of these spin variables, and need not inquire further into their representation as matrices, or otherwise, except as curiosity may dictate.[8]

For the simplification of the algebraic manipulations with the spin variables it is convenient to tabulate some further relations which they obey as a result of Eqs. (1) and (1'). If $\mathbf{n}$ and $\mathbf{m}$ are two ordinary unit vectors, the following relations can be proved readily by writing out both members:

$$\begin{aligned}
&(\mathbf{n}\cdot\boldsymbol{\sigma})^2=1,\\
&\boldsymbol{\sigma}(\mathbf{n}\cdot\boldsymbol{\sigma})=\mathbf{n}+i(\mathbf{n}\times\boldsymbol{\sigma}),\\
&(\mathbf{n}\cdot\boldsymbol{\sigma})\boldsymbol{\sigma}=\mathbf{n}-i(\mathbf{n}\times\boldsymbol{\sigma}),\\
&(\mathbf{m}\cdot\boldsymbol{\sigma})(\mathbf{n}\cdot\boldsymbol{\sigma})=(\mathbf{m}\cdot\mathbf{n})+i(\mathbf{m}\times\mathbf{n})\cdot\boldsymbol{\sigma},\\
&(\mathbf{m}\cdot\boldsymbol{\sigma})\boldsymbol{\sigma}(\mathbf{n}\cdot\boldsymbol{\sigma})=\mathbf{n}(\mathbf{m}\cdot\boldsymbol{\sigma})+\mathbf{m}(\mathbf{n}\cdot\boldsymbol{\sigma})\\
&\qquad\qquad -i(\mathbf{m}\times\mathbf{n})-(\mathbf{m}\cdot\mathbf{n})\boldsymbol{\sigma}.
\end{aligned} \tag{2}$$

Scalar and vector products are defined here exactly as in ordinary vector analysis, except that the components of the spin vector are not commutative among themselves.

In order to associate these spin quantities with rotations, we first introduce a convenient notation for the latter. Let O be the fixed point about which the rotations take place. A rotation about

[8] A fuller discussion may be found in ref. 7. The general theory will be found in F. Murnaghan, *The theory of group representations* (Johns Hopkins, 1938).

the line of a unit vector $\mathbf{n}$, through an angle θ, will be symbolized as $R(\mathbf{n}, \theta)$.

If $\mathbf{r}$ is the radius vector drawn from O to an arbitrary point P, then the radius vector $\mathbf{r}'$ to the point P' into which P is carried by the rotation will be written as

$$\mathbf{r}' = R(\mathbf{n}, \theta)\mathbf{r}. \tag{3}$$

The successive application of two rotations, $R(\mathbf{n}, \theta)$ and $R(\mathbf{m}, \varphi)$, is equivalent to a single rotation $R(\mathbf{l}, \psi)$ given by the expression

$$R(\mathbf{l}, \psi) = R(\mathbf{m}, \varphi)R(\mathbf{n}, \theta), \tag{4}$$

the order of application of the rotations being read from right to left.

With each rotation we now associate an expression in terms of spinors by the following correspondence:

$$R(\mathbf{n}, \theta) \sim [\cos \tfrac{1}{2}\theta - i(\mathbf{n}\cdot\boldsymbol{\sigma}) \sin \tfrac{1}{2}\theta], \tag{5}$$

where it is implied that the algebraic properties of the expressions in the brackets are identical with those of the rotations to which they correspond.[9] In order to obviate the fact that the addition of an angle 2π to the angle θ does not alter the rotation, but changes the sign of the bracketed expression, we shall make the convention that all rotation angles are less than 2π.

2. The Basic Formulas

The first use to which we shall put this correspondence is in a derivation of the formula for the composition of rotations, which is expressed by Eq. (4). Writing this relation out in full, we have

$$R(\mathbf{l}, \psi) \sim [\cos \tfrac{1}{2}\varphi - i(\mathbf{m}\cdot\boldsymbol{\sigma}) \sin \tfrac{1}{2}\varphi] \times [\cos \tfrac{1}{2}\theta - i(\mathbf{n}\cdot\boldsymbol{\sigma}) \sin \tfrac{1}{2}\theta]. \tag{6}$$

This expression can now be multiplied out and readily reduced to the standard form of Eq. (5) by making use of Eqs. (2), from which it is found that the properties of the resultant rotation $R(\mathbf{l}, \psi)$ are expressed in the formulas

$$\cos \tfrac{1}{2}\psi = \cos \tfrac{1}{2}\varphi \cos \tfrac{1}{2}\theta - (\mathbf{m}\cdot\mathbf{n}) \sin \tfrac{1}{2}\varphi \sin \tfrac{1}{2}\theta, \tag{7}$$

$$\mathbf{l} \sin \tfrac{1}{2}\psi = (\mathbf{m}\times\mathbf{n}) \sin \tfrac{1}{2}\varphi \sin \tfrac{1}{2}\theta + \mathbf{m} \sin \tfrac{1}{2}\varphi \cos \tfrac{1}{2}\theta + \mathbf{n} \sin \tfrac{1}{2}\theta \cos \tfrac{1}{2}\varphi. \tag{7'}$$

These equations furnish us with the basic *multiplication law* of the rotation group.

It follows at once from an inspection of Eq. (7) that the rotation angle is independent of the order of performing the separate rotations. From Eq. (7') we see that the direction of the axis of the resultant rotation depends on the order, since an interchange of the order changes the sign of the first term.

In deriving the analytic relation implied in Eq. (3), we shall employ a method that is of considerable interest for its use in crystal structure theory. We first define two rotations as being of the same *class* if they have the same rotation angle, though they may have different axes. We wish to establish an analytic relationship between rotations of the same class.

In Fig. 1 let $\mathbf{l}$ and $\mathbf{m}$ be the axes of two rotations through the angle φ, and let $R(\mathbf{n}, \theta)$ be any rotation that carries $\mathbf{l}$ into $\mathbf{m}$. We can now find the rotation $R(\mathbf{m}, \varphi)$ by the following process: we first rotate $\mathbf{m}$ into $\mathbf{l}$ by the rotation inverse to $R(\mathbf{n}, \theta)$, perform the rotation $R(\mathbf{l}, \varphi)$, and finally rotate $\mathbf{l}$ back into $\mathbf{m}$ by $R(\mathbf{n}, \theta)$. Since the line of the unit vector $\mathbf{m}$ occupies the same initial and final position, the result must be equivalent to a rotation about $\mathbf{m}$.

To carry out this procedure analytically, we first note that the inverse of the rotation $R(\mathbf{n}, \theta)$ is given by

$$[R(\mathbf{n}, \theta)]^{-1} = R(\mathbf{n}, -\theta) = R(-\mathbf{n}, \theta),$$

so that the foregoing sequence becomes

$$R(\mathbf{m}, \varphi) = R(\mathbf{n}, \theta)R(\mathbf{l}, \varphi)R(-\mathbf{n}, \theta).$$

On working this out, by a double application of Eqs. (7) and (7') we verify first of all that the angles of rotation about $\mathbf{m}$ and $\mathbf{l}$ actually are the same, and secondly we find the relation

$$\mathbf{m} = \mathbf{l} \cos\theta + \mathbf{n}(\mathbf{n}\cdot\mathbf{l}) 2 \sin^2 \tfrac{1}{2}\theta + (\mathbf{n}\times\mathbf{l}) \sin\theta. \tag{8}$$

But we now recognize that this is simply an operation in which the vector $\mathbf{l}$ is carried into $\mathbf{m}$ according to the formula

$$\mathbf{m} = R(\mathbf{n}, \theta)\mathbf{l}.$$

If now $\mathbf{r}$ is any vector along the axis of $\mathbf{l}$, its position after the rotation $R(\mathbf{n}, \theta)$ will be that of the vector $\mathbf{r}'$ along $\mathbf{m}$, so that the desired relation between $\mathbf{r}$ and $\mathbf{r}'$, according to Eq. (3), follows at once from Eq. (8) as

$$\mathbf{r}' = \mathbf{r} \cos\theta + \mathbf{n}(\mathbf{n}\cdot\mathbf{r}) 2 \sin^2 \tfrac{1}{2}\theta + (\mathbf{n}\times\mathbf{r}) \sin\theta. \tag{9}$$

[9] By making use of the first of Eqs. (2), this expression can be put in the form of an exponential function, since

$$\exp\{-i\tfrac{1}{2}(\mathbf{n}\cdot\boldsymbol{\sigma})\theta\} = \sum_{k=0}^{\infty} [-\tfrac{1}{2}i(\mathbf{n}\cdot\boldsymbol{\sigma})\theta]^k/k! = \cos \tfrac{1}{2}\theta - i(\mathbf{n}\cdot\boldsymbol{\sigma}) \sin \tfrac{1}{2}\theta;$$

but this is a purely formal correspondence, the form given in Eq. (5) being the most useful for manipulation.

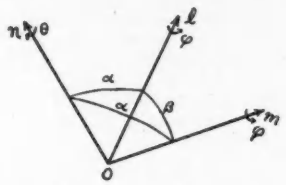

FIG. 1. The association of rotations into classes.

Equations (7), (7′) and (9) form the foundation for the study of the group of rotations about a fixed point.

With the help of Eq. (8) we can establish a useful formula giving the required angle of rotation θ in terms of the angles α and β shown in Fig. 1. For since $\mathbf{m}\cdot\mathbf{l}=\cos\beta$ and $\mathbf{n}\cdot\mathbf{m}=\mathbf{n}\cdot\mathbf{l}=\cos\alpha$, we readily find from Eq. (8) that

$$\sin\tfrac{1}{2}\theta=\sin\tfrac{1}{2}\beta\csc\alpha. \tag{10}$$

3. The Cartesian Transformation

As an application of Eq. (9) we can express the transformation equations of the Cartesian components of the point P in terms of the direction cosines of the rotation axis $\mathbf{n}$ and the angle of rotation θ. On choosing a set of reference axes $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ we have: $\mathbf{n}=n_1\mathbf{i}+n_2\mathbf{j}+n_3\mathbf{k}$; $\mathbf{r}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$; $\mathbf{r}'=x'\mathbf{i}+y'\mathbf{j}+z'\mathbf{k}$. On substitution in Eq. (9), we find

$$\begin{aligned}x'=x(\cos\theta+2n_1^2\sin^2\tfrac{1}{2}\theta)+y(2n_1n_2\sin^2\tfrac{1}{2}\theta\\ -n_3\sin\theta)+z(2n_1n_3\sin^2\tfrac{1}{2}\theta+n_2\sin\theta),\end{aligned}$$

with similar relations for y' and z'. These rarely quoted relations seem to have been first found by Euler.[10]

4. The Eulerian Rotation

The extensive use of Eulerian angles in the dynamical theory of rigid body motions makes it instructive to determine the rotation involved in the use of these coordinates. In Fig. 2 let $(\mathbf{I}, \mathbf{J}, \mathbf{K})$ be a set of axes specifying the initial position, and $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ be a set giving the final position. The rotation which brings the first into the second set can be specified by a succession of three rotations:

(i) a rotation about $\mathbf{K}$ through the angle b;
(ii) a rotation about the "line of nodes" OA through the angle a;
(iii) a rotation about $\mathbf{k}$ through the angle c.

[10] E. T. Benedikt, J. App. Phys. **15**, 613 (1944).

If the unit vector along OA is designated by $\mathbf{M}$, then the rotation which we seek is the resultant given by the formula

$$\mathrm{R}(a, b, c)\equiv\mathrm{R}(\mathbf{n}, \theta)=\mathrm{R}(\mathbf{k}, c)\mathrm{R}(\mathbf{M}, a)\mathrm{R}(\mathbf{K}, b).$$

From Fig. 2 we read off the relations

$$\begin{aligned}\mathbf{M}&=\mathbf{I}\cos b+\mathbf{J}\sin b=\mathbf{i}\cos c-\mathbf{j}\sin c,\\ \mathbf{k}&=\mathbf{I}\sin a\sin b-\mathbf{J}\sin a\cos b+\mathbf{K}\cos a.\end{aligned}$$

A double application of Eqs. (7) and (7′) shows the resultant rotation to be given by the expressions

$$\begin{aligned}\cos\tfrac{1}{2}\theta&=\cos\tfrac{1}{2}a\cos\tfrac{1}{2}(b+c),\\ \mathbf{n}\sin\tfrac{1}{2}\theta&=\mathbf{I}\sin\tfrac{1}{2}a\cos\tfrac{1}{2}(b-c)\\ &\quad+\mathbf{J}\sin\tfrac{1}{2}a\sin\tfrac{1}{2}(b-c)\\ &\quad+\mathbf{K}\cos\tfrac{1}{2}a\sin\tfrac{1}{2}(b+c)\\ &=\mathbf{i}\sin\tfrac{1}{2}a\cos\tfrac{1}{2}(b-c)\\ &\quad+\mathbf{j}\sin\tfrac{1}{2}a\sin\tfrac{1}{2}(b-c)\\ &\quad+\mathbf{k}\cos\tfrac{1}{2}a\sin\tfrac{1}{2}(b+c).\end{aligned} \tag{11}$$

Since this rotation takes $\mathbf{I}$ into $\mathbf{i}$, we have $(\mathbf{n}\cdot\mathbf{I})=(\mathbf{n}\cdot\mathbf{i})$, with similar relations for the other axes; it follows that vectors lying along the rotation axes have identical components in the two frames of reference vectors, as is exemplified in Eq. (11).

The inverse of this rotation is given by

$$[\mathrm{R}(a, b, c)]^{-1}=\mathrm{R}(-a, -c, -b).$$

5. Infinitesimal Rotations

In view of the predominant role played in mechanics by the infinitesimal rotations, it is instructive to study their properties as a limiting case of the results for finite rotations. If we consider the angles θ, φ, ψ to be infinitesimals in Eq. (7′), we find that to first orders of small quantities,

$$\mathbf{l}\psi=\mathbf{m}\varphi+\mathbf{n}\theta,$$

which proves the well-known result for the compounding of infinitesimal rotations by vector addition. The noncommutative term in the right-hand member of Eq. (7′) becomes of second order and so drops out in this approximation.

A more subtle and extensive series of problems arises if one considers the question of the "differentiation" of rotations, that is, the combination of an infinitesimal with a finite rotation.

As a preliminary remark, let us consider again the specification of a rotation by means of Euler's angles, as in Fig. 2. Having rotated the system from the first to the second position as described in SEC. 4, let us perform an additional

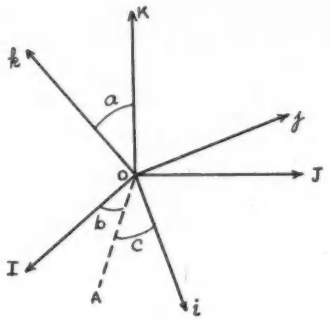

FIG. 2. The Eulerian rotation.

small rotation specified by making small changes da, db, dc in the angles. It is clear directly from Fig. 2 that this is done by making the small rotations about the lines of the vectors **K**, **M** and **k**, so that, applying the vector addition formula, the resultant rotation is given by the expression

$$\mathbf{m}\,d\varphi=\mathbf{K}db+\mathbf{M}da+\mathbf{k}dc. \qquad (12)$$

By adding to this the idea that the angular changes take place in time, we arrive at the usual expression for the angular velocity vector,

$$\omega=\mathbf{m}\,d\varphi/dt=\mathbf{K}db/dt+\mathbf{M}da/dt+\mathbf{k}dc/dt.$$

Let us now consider this process from a more general point of view. We suppose that the rotation from the first to the second position is brought about by the finite rotation $R(\mathbf{n}, \theta)$, and that the system is then given a further small rotation which brings it to a final position that is obtainable from the initial position by the application of the finite rotation $R(\mathbf{n}+d\mathbf{n}, \theta+d\theta)$. If we designate the infinitesimal rotation which does this as $R(\mathbf{m}, d\varphi)$, we have the relation

$$R(\mathbf{n}+d\mathbf{n}, \theta+d\theta)=R(\mathbf{m}, d\varphi)R(\mathbf{n}, \theta),$$

which can be solved at once by multiplication on the right of both members by $R(-\mathbf{n}, \theta)$ to give

$$R(\mathbf{m}, d\varphi)=R(\mathbf{n}+d\mathbf{n}, \theta+d\theta)R(-\mathbf{n}, \theta).$$

On working out this relation by means of Eqs. (7) and (7′), we find that the infinitesimal rotation is specified by the relation

$$\mathbf{m}\,d\varphi=(\mathbf{n}\times d\mathbf{n})2\sin^2\tfrac{1}{2}\theta+d\mathbf{n}\sin\theta+\mathbf{n}d\theta. \qquad (13)$$

Here $d\theta$ is any arbitrary small change in the rotation angle, while $d\mathbf{n}$ is an arbitrary small alteration in the direction of the axis of rotation, which is limited only by the condition that it must be perpendicular to **n**; that is, $\mathbf{n}\cdot d\mathbf{n}=0$, a condition which follows from the fact that both **n** and $\mathbf{n}+d\mathbf{n}$ are unit vectors.

By expressing $d\mathbf{n}$ and $d\theta$ in terms of da, db, dc from Eq. (11), it is possible to reduce Eq. (13) to Eq. (12), but the calculation is rather laborious.

This is not the only way in which we can use an infinitesimal rotation as the bridge between two neighboring finite rotations $R(\mathbf{n}, \theta)$ and $R(\mathbf{n}+d\mathbf{n}, \theta+d\theta)$. For, instead of applying the infinitesimal rotation *after* $R(\mathbf{n}, \theta)$, let us start out from the initial position by applying such an infinitesimal rotation $R(\mathbf{l}, d\psi)$ that when this is followed by $R(\mathbf{n}, \theta)$ we shall attain the same final position as is given by $R(\mathbf{n}+d\mathbf{n}, \theta+d\theta)$. This sequence is given by

$$R(\mathbf{n}+d\mathbf{n}, \theta+d\theta)=R(\mathbf{n}, \theta)R(\mathbf{l}, d\psi).$$

On solving for the infinitesimal rotation in the same manner as before, we find

$$\mathbf{l}\,d\psi=-(\mathbf{n}\times d\mathbf{n})2\sin^2\tfrac{1}{2}\theta+d\mathbf{n}\sin\theta+\mathbf{n}d\theta, \qquad (14)$$

which differs in its first term from the rotation of Eq. (13).

The two procedures employed in obtaining Eqs. (13) and (14) may be defined as *post-differentiation* and *pre-differentiation*, respectively, of $R(\mathbf{n}, \theta)$. Since the former leads directly to the angular velocity vector used in mechanics, it has the preferred physical interpretation; but in a mathematical sense the two forms of differentiation are on equivalent footings.

This dual method of associating infinitesimal transformations with neighboring finite transformations has been extended into an elegant mode of treatment of the general theory of finite continuous groups by the eminent French mathematician, Elie Cartan.[11]

[11] E. Cartan, *La théorie des groupes finis et continus* (Gauthier-Villars, 1937).

No more fiction for us: we calculate; but that we may calculate, we had to make fiction first.—NIETZSCHE.

Action and Reaction Between Moving Charges

LEIGH PAGE AND NORMAN I. ADAMS, JR.
Yale University, New Haven, Connecticut

ONE of the fundamental axioms of Newtonian dynamics asserts that the forces of action and reaction between two particles are equal in magnitude and opposite in direction. Its two most important consequences are (i) the law of conservation of the *linear* momentum of a group of particles subject to no external forces, and (ii) the conclusion that the forces between the particles play no part in determining the motion of the center of mass of a group of particles. The law of action and reaction is not sufficient, however, to prove the law of conservation of the *angular* momentum of a group of particles subject to no external torques; we must assume in addition that the forces between two particles are along the line joining them.

The laws of conservation of linear and of angular momentum are so well verified by experiments in mechanics that we cannot doubt the validity of the axioms on which they rest, at least insofar as *time-average* forces are concerned. Yet we believe that the atoms of which matter is constructed are composed of rapidly moving charged particles, and the electromagnetic forces between moving charged particles satisfy, in general, neither the condition that the forces between two particles are along the line joining them nor the condition that these forces are equal in magnitude and oppositely directed. It is the object of this paper to discuss the forces and torques exerted by one charged particle on another with special reference to the law of action and reaction. We shall employ Gaussian units (electrostatic units for electric quantities, including current, and electromagnetic units for magnetic quantities). Then the power to which $1/c$ appears as a factor (c is the ratio of the electromagnetic to the electrostatic unit of charge), determines the order of magnitude of a quantity. Thus terms in a series that do not contain $1/c$ will be designated as of zero order, those in $1/c$ as of the first order of smallness, and those in $(1/c)^2$ as of the second order of smallness. In our analysis we shall retain all terms through the second order, but none of higher order.

TWO CURRENT ELEMENTS

The magnetic field intensity $\mathbf{H}$ caused by a current element—an element of length $\Delta\boldsymbol{\lambda}$ of a linear circuit carrying a current i—at a field point P at a vector distance $\mathbf{r}$ from the current element, is generally specified by the formula

$$\mathbf{H}=i\Delta\boldsymbol{\lambda}\times\mathbf{r}/cr^3. \tag{1}$$

While this expression is often referred to as Ampère's law, it is not the expression originally proposed by Ampère. In fact, the original Ampère's law,[1] while like Eq. (1) in that it yields correct results for *constant* currents in *closed* circuits, does not specify correctly even the first-order magnetic field of a current element. This first-order field is given correctly by Eq. (1). However, we know today that Eq. (1) is merely the first term of an infinite series[2] in powers of $1/c$. Nevertheless, this term alone yields an exceedingly accurate value of the field, owing to the fact that the next term in the series happens to have the coefficient zero, and therefore the error in Eq. (1) is of the order of $(1/c)^3$. Since we are interested here only in terms of the second and lower orders, we can employ Eq. (1) without modification.

A current element is usually thought of as a charge e moving along the circuit so as to constitute a current, associated with a charge $-e$ at rest, so that the total charge is zero. For the present, we shall dispense with the stationary charge $-e$ and confine our attention to a single charge e moving with velocity $\mathbf{v}$. Then, if ρ_λ is the charge per unit length of the circuit, $e=\rho_\lambda\Delta\lambda$ and $i=\rho_\lambda v$. Hence $e\mathbf{v}=i\Delta\boldsymbol{\lambda}$, and we may write Eq. (1) in the form

$$\mathbf{H}=e\mathbf{v}\times\mathbf{r}/cr^3. \tag{2}$$

To calculate the force exerted by one moving charge on another we also need the formula for the electric field intensity. The electric field of a moving charge differs from the simple inverse-square field of a charge at rest, the intensity at the field point P being specified by

$$\mathbf{E}=\frac{e\mathbf{r}}{r^3}-\frac{e}{2c^2}\left\{\frac{\mathbf{f}}{r}+\left(\frac{\mathbf{f}\cdot\mathbf{r}}{r^3}-\frac{v^2}{r^3}+3\frac{\overline{\mathbf{v}\cdot\mathbf{r}}^2}{r^5}\right)\mathbf{r}\right\} \tag{3}$$

[1] See Maxwell, *Electricity and magnetism*, Part IV, chap. II, for a detailed discussion of the original Ampère's law.
[2] Page and Adams, *Electrodynamics*, p. 175.

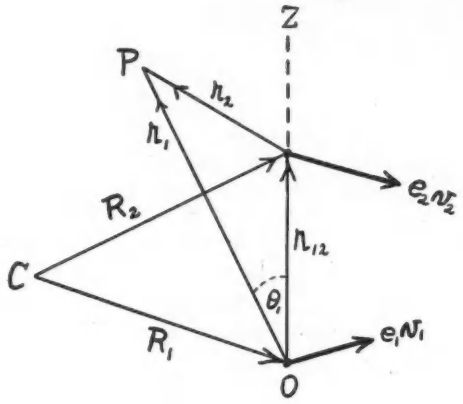

FIG. 1. Two moving charges.

through second-order terms.[3] Here $\mathbf{f}$ is the acceleration of the charge.

Now we are ready to calculate the forces between two moving charges, $e_1\mathbf{v}_1$ and $e_2\mathbf{v}_2$ (Fig. 1). Let $\mathbf{r}_{12}$ be the vector distance of e_2 from e_1, and $\mathbf{r}_{21}[=-\mathbf{r}_{12}]$ that of e_1 from e_2. First we calculate the magnetic forces $\mathbf{F}_1{}^H$ on the current element $e_1\mathbf{v}_1$ and $\mathbf{F}_2{}^H$ on $e_2\mathbf{v}_2$. Using Eq. (2) and the familiar formula,

$$\mathbf{F}^H=\frac{1}{c}e\mathbf{v}\times\mathbf{H},$$

for the force on a current element $e\mathbf{v}$ in a magnetic field $\mathbf{H}$, we find

$$\mathbf{F}_1{}^H=\frac{e_1\mathbf{v}_1}{c}\times\left\{\frac{e_2\mathbf{v}_2\times\mathbf{r}_{21}}{cr^3}\right\}=\frac{e_1e_2}{c^2}\frac{\mathbf{v}_1\times(\mathbf{v}_2\times\mathbf{r}_{21})}{r^3},$$

$$\mathbf{F}_2{}^H=\frac{e_2\mathbf{v}_2}{c}\times\left\{\frac{e_1\mathbf{v}_1\times\mathbf{r}_{12}}{cr^3}\right\}=\frac{e_1e_2}{c^2}\frac{\mathbf{v}_2\times(\mathbf{v}_1\times\mathbf{r}_{12})}{r^3},$$

where $r[=|\mathbf{r}_{12}|=|\mathbf{r}_{21}|]$ is the distance between the charges.

If the magnetic forces obeyed the law of action and reaction, $\mathbf{F}_1{}^H+\mathbf{F}_2{}^H$ would vanish. But, instead,

$$\mathbf{F}_1{}^H+\mathbf{F}_2{}^H=\frac{e_1e_2}{c^2r^3}(\mathbf{v}_2\times\mathbf{v}_1)\times\mathbf{r}_{12}, \tag{4}$$

which vanishes only when (a) $\mathbf{v}_2$ is parallel to $\mathbf{v}_1$, or (b) $\mathbf{v}_1$ and $\mathbf{v}_2$ are both perpendicular to $\mathbf{r}_{12}$. So, except in these special cases, the magnetic forces between two moving charges do not constitute an equal and opposite pair satisfying the law of action and reaction, but have a vector resultant different from zero.

Next we calculate the electric forces $\mathbf{F}_1{}^E$ on $e_1\mathbf{v}_1$ and $\mathbf{F}_2{}^E$ on $e_2\mathbf{v}_2$ from Eq. (3) and the formula $\mathbf{F}^E=e\mathbf{E}$. The zero-order terms form an equal and opposite pair satisfying the law of action and reaction, but the second-order terms do not. In fact, we find for the resultant electric force,

$$\mathbf{F}_1{}^E+\mathbf{F}_2{}^E=\frac{e_1e_2}{2c^2}\left[-\frac{\mathbf{f}_1+\mathbf{f}_2}{r}+\left\{-(\mathbf{f}_1+\mathbf{f}_2)\cdot\mathbf{r}_{12}+v_1{}^2-v_2{}^2-3\frac{\overline{\mathbf{v}_1\cdot\mathbf{r}_{12}}{}^2}{r^2}+3\frac{\overline{\mathbf{v}_2\cdot\mathbf{r}_{12}}{}^2}{r^2}\right\}\frac{\mathbf{r}_{12}}{r^3}\right], \tag{5}$$

which does not vanish in general, although it is zero in the special case $\mathbf{f}_2=-\mathbf{f}_1$, $\mathbf{v}_2=\pm\mathbf{v}_1$.

Evidently the total resultant force through second-order terms, obtained by adding Eqs. (4) and (5), does not vanish in general. Therefore the law of action and reaction does not hold for two moving charges. This conclusion, however, should occasion no surprise, for it is a well-known consequence of the electromagnetic equations that the so-called mechanical momentum of a system of charged particles subject to no external forces does not remain constant in time. On the contrary, the conservation law applies only to the *sum* of the mechanical and the electromagnetic momentums. This statement applies both to linear and to angular momentum. Thus a photon emitted from the sun subtracts from the mechanical momentum of the latter an amount equal to its own electromagnetic momentum.

It is, therefore, of interest to calculate the electromagnetic linear momentum of the field of the two moving charges under consideration, in order to show that its time-rate of increase, when added to the sum of Eqs. (4) and (5), gives zero resultant.

In Gaussian units the electromagnetic linear momentum per unit volume is given[4] by $(1/4\pi c)\mathbf{E}\times\mathbf{H}$. Consider a field point P (Fig. 1) at a vector distance $\mathbf{r}_1$ from $e_1\mathbf{v}_1$ and $\mathbf{r}_2$ from $e_2\mathbf{v}_2$. Let $\mathbf{E}_1$, $\mathbf{H}_1$ be the field intensities caused by the first current element, and $\mathbf{E}_2$, $\mathbf{H}_2$ those caused by the second. Then the electromagnetic linear momentum per unit volume is

$$\frac{1}{4\pi c}\{\mathbf{E}_1\times\mathbf{H}_1+\mathbf{E}_1\times\mathbf{H}_2+\mathbf{E}_2\times\mathbf{H}_1+\mathbf{E}_2\times\mathbf{H}_2\}.$$

[3] Reference 2, p. 175.

[4] Reference 2, p. 272.

Evidently the two terms in which the subscripts are the same remain constant if the velocities of the two charges do not change, and are responsible for the electromagnetic masses of the individual charges in accelerated motion. As we are interested here only in the interaction of the one current element with the other, we need retain only the mutual terms, and calculate the portion of the electromagnetic linear momentum per unit volume represented by

$$\mathbf{g}_l=\frac{1}{4\pi c}\{\mathbf{E}_1\times\mathbf{H}_2+\mathbf{E}_2\times\mathbf{H}_1\}. \tag{6}$$

Since Eq. (2) for $\mathbf{H}$ is of the first order, we need retain only the first term in Eq. (3) for $\mathbf{E}$ in order to obtain $\mathbf{g}_l$ through second-order terms. Thus

$$\mathbf{g}_l=\frac{e_1e_2}{4\pi c^2r_1{}^3r_2{}^3}\{\mathbf{r}_1\times(\mathbf{v}_2\times\mathbf{r}_2)+\mathbf{r}_2\times(\mathbf{v}_1\times\mathbf{r}_1)\}. \tag{7}$$

To obtain the total mutual electromagnetic linear momentum $\mathbf{G}_l$ we must integrate this expression over all space. To do this easily we expand the triple vector products and introduce spherical coordinates r_1, θ_1, ϕ with O as origin and OZ as polar axis. From the trigonometric relation $r_2{}^2=r^2-2rr_1\cos\theta_1+r_1{}^2$ we find,[5] for constant r_1, that $r_2\mathrm{d}r_2=r_1r\sin\theta_1\mathrm{d}\theta_1$. Hence the volume element $\mathrm{d}\tau=r_1{}^2\sin\theta_1\mathrm{d}r_1\mathrm{d}\theta_1\mathrm{d}\phi$ may be written $\mathrm{d}\tau=(r_1r_2/r)\mathrm{d}r_1\mathrm{d}r_2\mathrm{d}\phi$ in terms of r_1, r_2, ϕ as variables. For $r_1<r$ the limits of r_2 are $r-r_1$ and $r+r_1$, whereas for $r_1>r$, r_2 goes from r_1-r to r_1+r. The integration, which involves no difficulties, yields the result

$$\mathbf{G}_l=\frac{e_1e_2}{2c^2}\left\{\left(\frac{\mathbf{v}_1}{r}+\frac{\mathbf{v}_1\cdot\mathbf{r}_{12}\mathbf{r}_{12}}{r^3}\right)+\left(\frac{\mathbf{v}_2}{r}+\frac{\mathbf{v}_2\cdot\mathbf{r}_{12}\mathbf{r}_{12}}{r^3}\right)\right\}. \tag{8}$$

The reader can show easily that the time derivative of $\mathbf{G}_l$, when added to the sum of Eqs. (4) and (5), gives zero resultant, thus verifying the statement that the sum of the mechanical and electromagnetic linear momentums remains constant in time. The fact that the electromagnetic momentum changes when $\mathbf{r}_{12}$ is not constant (that is, when $\mathbf{v}_2$ is not equal to $\mathbf{v}_1$) *requires* that the forces of action and reaction between the two moving charges should *not* be equal in magnitude and oppositely directed. Otherwise the total momentum could *not* remain constant in time.

Next we calculate the torque or moment about an arbitrary point C (Fig. 1) of the force $\mathbf{F}_1{}^H+\mathbf{F}_1{}^E$ on $e_1\mathbf{v}_1$ and of the force $\mathbf{F}_2{}^H+\mathbf{F}_2{}^E$ on $e_2\mathbf{v}_2$. Denoting the position vectors of the two charges relative to C by $\mathbf{R}_1$ and $\mathbf{R}_2$, we have for the sum of the torques of the magnetic forces,

$$\mathbf{R}_1\times\mathbf{F}_1{}^H+\mathbf{R}_2\times\mathbf{F}_2{}^H=\frac{e_1e_2}{c^2r^3}[-\mathbf{R}_1\times\{\mathbf{v}_1\times(\mathbf{v}_2\times\mathbf{r}_{12})\}+\mathbf{R}_2\times\{\mathbf{v}_2\times(\mathbf{v}_1\times\mathbf{r}_{12})\}], \tag{9}$$

and for the sum of the torques of the electric forces,

$$\mathbf{R}_1\times\mathbf{F}_1{}^E+\mathbf{R}_2\times\mathbf{F}_2{}^E=\frac{e_1e_2}{2c^2}\left[\mathbf{R}_1\times\left\{-\frac{\mathbf{f}_1}{r}+\left(-\mathbf{f}_1\cdot\mathbf{r}_{12}+v_1{}^2-3\frac{\overline{\mathbf{v}_1\cdot\mathbf{r}_{12}}{}^2}{r^2}\right)\frac{\mathbf{r}_{12}}{r^3}\right\}+\mathbf{R}_2\times\left\{-\frac{\mathbf{f}_2}{r}+\left(-\mathbf{f}_2\cdot\mathbf{r}_{12}-v_2{}^2+3\frac{\overline{\mathbf{v}_2\cdot\mathbf{r}_{12}}{}^2}{r^2}\right)\frac{\mathbf{r}_{12}}{r^3}\right\}\right]. \tag{10}$$

Evidently the resultant torque, obtained by adding Eqs. (9) and (10), does not vanish in general, indicating that the mechanical angular momentum of the pair of moving charges does not remain constant in time. We conclude that this lack of balance is due to the presence of a changing electromagnetic angular momentum, which we proceed to calculate.

Relative to the point C the position vector of the field point P is $\mathbf{R}_1+\mathbf{r}_1=\mathbf{R}_2+\mathbf{r}_2$. Hence the mutual electromagnetic angular momentum per unit volume is

$$\mathbf{g}_a=(\mathbf{R}_1+\mathbf{r}_1)\times\mathbf{g}_l.$$

Integrating over all space, we find for the total mutual electromagnetic angular momentum about C,

$$\mathbf{G}_a=\mathbf{R}_1\times\mathbf{G}_l+\frac{e_1e_2}{2c^2}\frac{\mathbf{r}_{12}\times\mathbf{v}_1}{r},$$

[5] Of course r is not a variable for this integration.

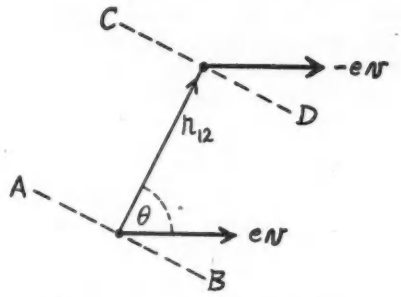

FIG. 2. The Trouton-Noble experiment.

which can be put in the more symmetrical form,

$$\mathbf{G}_a=\frac{e_1e_2}{2c^2}\left\{\mathbf{R}_2\times\left(\frac{\mathbf{v}_1}{r}+\frac{\mathbf{v}_1\cdot\mathbf{r}_{12}\mathbf{r}_{12}}{r^3}\right)+\mathbf{R}_1\times\left(\frac{\mathbf{v}_2}{r}+\frac{\mathbf{v}_2\cdot\mathbf{r}_{12}\mathbf{r}_{12}}{r^3}\right)\right\}. \quad (11)$$

Comparing with Eq. (8) we see that the portion of the linear momentum involving the velocity $\mathbf{v}_1$ of the first particle is to be considered as located at the second particle, and *vice versa*. The time derivative of $\mathbf{G}_a$, added to the torques of the forces on the two current elements specified by the sum of Eqs. (9) and (10), gives zero resultant, in accord with the law of conservation of the *sum* of the mechanical and electromagnetic angular momentums. Neither one of these two types of angular momentum alone is constant in time in the general case, even though no external torques are operating.

It is worthy of note that both $\mathbf{G}_l$ and $\mathbf{G}_a$ are of the second order. This is because $\mathbf{H}$ contains no zero-order term. Therefore the law of action and reaction fails to hold only by terms of the second or higher orders.

Finally, we must extend our analysis to cover the case of two conventional current elements, in each of which there is associated with the charge e moving with velocity $\mathbf{v}$ an equal and opposite stationary charge $-e$. Since the net charge of each current element is zero, the electric force on it vanishes. Therefore the entire resultant force is given by Eq. (4).

The zero-order electric field of each current element is zero at the instant when the moving charge coincides with the stationary charge of opposite sign. At this instant the mutual electromagnetic linear momentum $\mathbf{G}_l$ vanishes. However, $\mathbf{G}_l$ does not remain zero as these charges separate, and therefore the time derivative of $\mathbf{G}_l$ does not vanish, in general. For let us designate the coordinates of the two current elements by (x_1, y_1, z_1), (x_2, y_2, z_2), respectively, so that

$$\mathbf{r}_{12}=\mathbf{i}(x_2-x_1)+\mathbf{j}(y_2-y_1)+\mathbf{k}(z_2-z_1)$$

and

$$r^2=(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2.$$

Now, when we differentiate the $\mathbf{r}_{12}$ and r appearing in Eq. (8), where both e_1 and e_2 are *moving* charges, we must differentiate both sets of coordinates. But in the case of the complete current elements which we are now discussing, the expression for the mutual electromagnetic linear momentum contains, in addition to the right-hand member of Eq. (8), the term

$$-\frac{e_1e_2}{2c^2}\left(\frac{\mathbf{v}_1}{r}+\frac{\mathbf{v}_1\cdot\mathbf{r}_{12}\mathbf{r}_{12}}{r^3}\right),$$

in which e_1 is a *moving* charge and $-e_2$ a *stationary* charge, and the term

$$-\frac{e_1e_2}{2c^2}\left(\frac{\mathbf{v}_2}{r}+\frac{\mathbf{v}_2\cdot\mathbf{r}_{12}\mathbf{r}_{12}}{r^3}\right),$$

in which e_2 is *moving* and $-e_1$ is *stationary*. In differentiating the first of these terms we must hold x_2, y_2, z_2 constant, and in differentiating the second, x_1, y_1, z_1. The final result is the same as if, in differentiating Eq. (8), we differentiated only the x_2, y_2, z_2 contained implicitly in $\mathbf{r}_{12}$ and r inside the first parentheses, and only the x_1, y_1, z_1 inside the second parentheses. This gives, for the time-rate of increase of $\mathbf{G}_l$, the negative of Eq. (4), confirming the conservation of the *sum* of the mechanical and the electromagnetic linear momentums.

APPLICATIONS

An example of the theory developed, which is of great historical interest, is illustrated in Fig. 2. Here we have two equal and opposite charges moving with the same constant velocity $\mathbf{v}$ in a direction *not* at right angles to the vector distance $\mathbf{r}_{12}$ between them. Evidently $\mathbf{G}_l$ is constant in time, and therefore the forces between the current elements are equal and opposite. This is not true, however, of the torques, for

$$\frac{d\mathbf{G}_a}{dt}=-\frac{e^2}{c^2}\frac{\mathbf{v}\cdot\mathbf{r}_{12}\mathbf{v}\times\mathbf{r}_{12}}{r^3},$$

and therefore there exists an equal and opposite torque on the moving charges, to wit,

$$\mathfrak{L}=\frac{e^2v^2}{c^2r}\cos\theta\sin\theta \qquad (12)$$

in the sense of increasing θ.

The arrangement in Fig. 2 is effectively that used in the Trouton-Noble[6] experiment, which excited much interest during the last generation, the charges being spread over the positively charged plate AB and the negatively charged plate CD of a parallel plate condenser. Trouton and Noble concluded from Eq. (12) that a charged condenser carried along by the earth in its motion around the sun would turn until its plates are parallel to the direction of motion, a phenomenon which is obviously at variance with the relativity principle. The fallacy in this reasoning lies in the neglect of the torque of the forces exerted on the condenser plates by the insulating separators necessary to keep the plates from approaching each other under their mutual electric attraction. The relativity principle, completely confirmed by the null result of the experiment, demands that these forces show the same aberration as the electromagnetic forces.

Next consider two charged particles, moving around their common center of mass under their mutual forces, such as the electron and the proton constituting a hydrogen atom. We shall suppose that the velocities attained are small enough so that we can neglect the variation of mass with velocity. The zero-order force is the inverse-square Coulomb force of attraction (or repulsion), and under such a force the two particles would revolve about the center of mass with constant angular momentum. We shall show that, when account is taken of second-order terms, the mechanical angular momentum does not remain constant, but, in the case of an (approximately) elliptical orbit, oscillates between a minimum at perihelion and a maximum at aphelion.

Take the center of mass as origin, and denote the position vectors of the two particles, of mass and charge m_1, e_1 and m_2, e_2, respectively, by $\mathbf{r}_1$ and $\mathbf{r}_2$. Let r be the distance between the two particles, and θ the angle in the plane of the orbit which the line joining them makes with some fixed line. From Eq. (11) we calculate the electromagnetic angular momentum about the

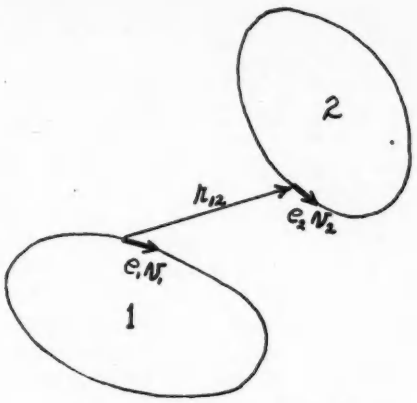

FIG. 3. Two complete circuits.

center of mass, finding

$$\mathbf{G}_a=\frac{e_1e_2}{2c^2}\left\{\frac{\mathbf{r}_1\times\mathbf{v}_2+\mathbf{r}_2\times\mathbf{v}_1}{r}\right\}.$$

But $|\mathbf{r}_1\times\mathbf{v}_2|=|\mathbf{r}_2\times\mathbf{v}_1|=-r_1r_2\dot{\theta}$, and, if μ is the reduced mass, $m_1r_1=m_2r_2=\mu r$. Hence

$$G_a=-\frac{e_1e_2}{c^2}\frac{r_1r_2}{r}\dot{\theta}=-\frac{e_1e_2\mu^2}{c^2m_1m_2}r\dot{\theta} \qquad (13)$$

in the sense of increasing θ. Now, so far as the zero-order motion is concerned, $r^2\dot{\theta}=h$ (a positive constant), and

$$G_a=-\frac{e_1e_2\mu^2}{c^2m_1m_2}\frac{h}{r}. \qquad (14)$$

If the charges are of opposite sign, the zero-order force is an attraction, and G_a is positive, attaining its maximum value when the particles are nearest together. Therefore, since the sum of the mechanical and the electromagnetic angular momentums must remain constant, the mechanical angular momentum is a minimum when the particles are closest together. In the case of an elliptical orbit, the mechanical angular momentum oscillates between a smallest value at perihelion and a largest value at aphelion. On the other hand, when the charges are of the same sign, so as to make the zero-order force a repulsion, G_a is negative, and therefore the electromagnetic angular momentum is an (algebraic) minimum and the mechanical angular momentum a maximum at the position of nearest approach. Since the zero-order mechanical angular mo-

[6] Proc. Roy. Soc. **72**, 132 (1903).

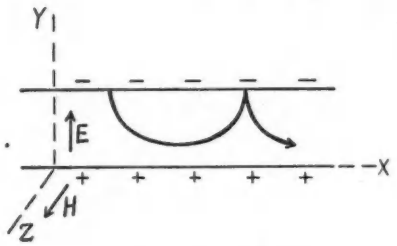

FIG. 4. Motion of electrons in perpendicular fields.

mentum is μh in either case, Eq. (14) may be regarded as expressing the electromagnetic angular momentum in terms of the zero-order mechanical angular momentum and the separation r of the particles. If we pass over from the two-body problem to the corresponding one-body problem by making one of the masses infinite, G_a vanishes, as expected.

As our next example we shall calculate the mutual electromagnetic linear momentum of two complete circuits *1* and *2*, Fig. 3, which we shall designate by the subscripts 1 and 2. Here we are concerned, not with the mutual momentum of two elements of the *same* circuit, but only with that of pairs of current elements such that one element of each pair is part of one circuit and the other is part of the other circuit.

At first we shall consider the case where the electric field of the moving charge is not neutralized by that of an equal stationary charge of opposite sign. Replacing e_1 and e_2 in Eq. (8) by de_1 and de_2, and using the relation $\mathbf{v}de = id\boldsymbol{\lambda}$, we obtain, after integrating Eq. (8) over circuit *2*,

$$d\mathbf{G}_l = \frac{1}{2c^2}\left\{de_1\mathbf{v}_1 \oint_2 \frac{de_2}{r} + de_1\mathbf{v}_1 \cdot \oint_2 \frac{\mathbf{r}_{12}\mathbf{r}_{12}de_2}{r^3} + de_1 i_2 \oint_2 \frac{d\boldsymbol{\lambda}_2}{r} + de_1 i_2 \oint_2 \frac{\mathbf{r}_{12}\cdot d\boldsymbol{\lambda}_2 \mathbf{r}_{12}}{r^3}\right\}.$$

Now, for the integration around circuit *2*, $d\boldsymbol{\lambda}_2 = d\mathbf{r}_{12}$. Hence, using the identity

$$d\left(\frac{\mathbf{r}_{12}}{r}\right) = \frac{d\mathbf{r}_{12}}{r} - \frac{\mathbf{r}_{12}\cdot d\mathbf{r}_{12}\mathbf{r}_{12}}{r^3},$$

we have

$$\oint_2 \frac{\mathbf{r}_{12}\cdot d\boldsymbol{\lambda}_2 \mathbf{r}_{12}}{r^3} = \oint \frac{d\boldsymbol{\lambda}_2}{r},$$

which enables us to combine the last two integrals in the expression for $d\mathbf{G}_l$. Integrating next over circuit *1*, and proceeding in the same manner with the first two integrals, we obtain finally for the electromagnetic linear momentum of the two circuits,

$$\mathbf{G}_l = \frac{i_1}{c^2}\oint_1 \oint_2 \frac{de_2 d\boldsymbol{\lambda}_1}{r} + \frac{i_2}{c^2}\oint_2 \oint_1 \frac{de_1 d\boldsymbol{\lambda}_2}{r}. \quad (15)$$

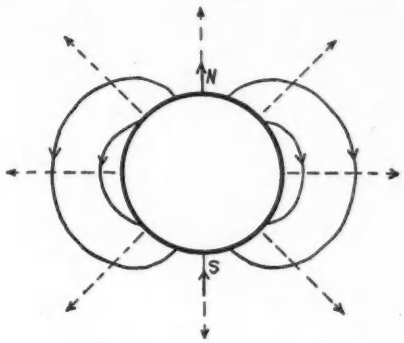

FIG. 5. Magnetized and charged sphere.

So long as the currents are constant, $\mathbf{G}_l$ is constant in time, and therefore the forces of action and reaction between the two circuits are equal in magnitude and opposite in direction.

Finally, if the moving charge in each circuit is associated with an equal charge of the opposite sign at rest, as in the conventional current circuit, the static electric field is everywhere zero, and consequently both $\mathbf{G}_l$ and its time derivative vanish. Therefore the law of action and reaction is valid for two such circuits carrying constant currents.

In closing we shall mention two other examples, the details of which will be found in our *Electrodynamics*. Consider a charged parallel plate condenser (Fig. 4), with plates perpendicular to the Y-axis, placed in a uniform magnetic field parallel to the Z-axis. Let electrons be liberated, say photoelectrically, from the upper negatively charged plate. Under the action of the combined fields they will describe cycloidal paths as indicated, and at the bottom of these paths will have acquired linear momentum in the X-direction. It can be shown easily that the mechanical linear momentum so acquired is exactly accounted for by the loss of electromagnetic linear momentum in the X-direction due to the weakening of the electric and magnetic fields by the displacement and motion of the electrons.

Finally, consider a sphere (Fig. 5) which is uniformly magnetized in the direction *SN* and at the same time carries a uniformly distributed positive electric charge on its surface. In Fig. 5 the lines of magnetic force are indicated by full lines, and the lines of electric force by broken lines. Evidently the electromagnetic field possesses angular momentum about the *SN*-axis in the positive sense. Now, suppose a negative ion, initially at rest, approaches the sphere under the electric attraction exerted by the latter. Owing to the deflecting force of the magnetic field, it will acquire angular momentum about the *SN*-axis in the positive sense, which it may communicate to the sphere upon impact. It can be shown without difficulty that the mechanical angular momentum so acquired is equal to the electromagnetic angular momentum lost as a result of the weakening of the field of the sphere by the attachment of the negative ion.

A Demonstration Laboratory for Advanced Dynamics

D. A. WELLS
University of Cincinnati, Cincinnati, Ohio

THE Lagrangian treatment of analytical dynamics has long been a powerful tool in the hands of the physicist. Owing to its many advantages in treating applied problems involving not only mechanical systems but electrical, electromechanical and others, the Lagrangian method is fast becoming an everyday tool in the hands of the engineer and applied scientist. It seems certain that in due time, perhaps very shortly, it will constitute an important part of the working equipment of every well-trained engineer. Thus the question as to how best to present this subject to advanced engineers and graduate students of physics presents a pedagogic problem that deserves careful consideration.

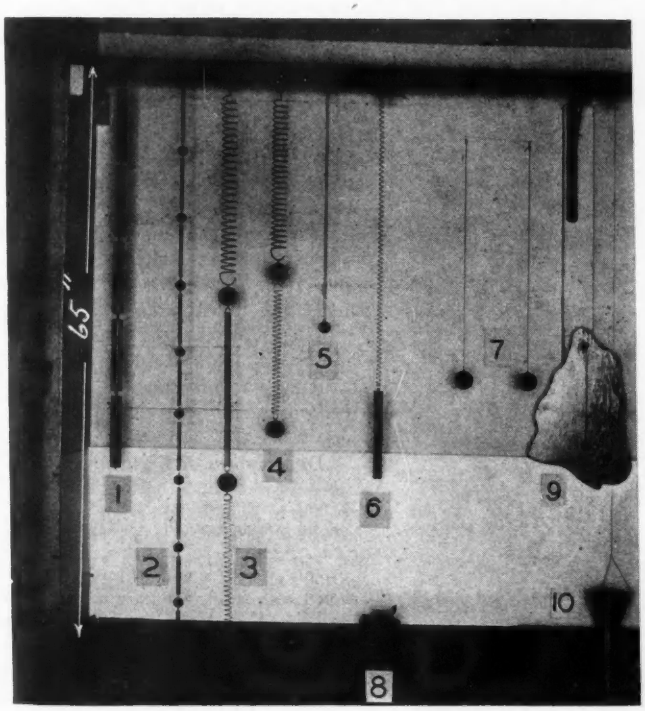

FIG. 1.

Now analytical dynamics can be treated from either of two quite different points of view:

(1) It can be regarded as an abstract mathematical subject in which generalized coordinates are just so many variable "parameters;" degrees of freedom and degrees of constraint merely connote certain mathematical relations; a nonholonomic system is one of those mathematical abortions in which occur certain differential relations that cannot be integrated; the Lagrangian function and generalized forces are but convenient mathematical definitions; and so on. Or

(2) it can be presented from a physical point of view in which care is taken to give the terms and mathematical relations employed simple and easy-to-recognize physi-

cal significance and where mathematics is given the position of a necessary and powerful language rather than that of the primary subject itself.

There is little doubt but that the second method is far better than the first, especially from the point of view of the applied scientist. Also there is little doubt but that the second method makes greater demands on both professor and students, for the obvious reason that it involves (*a*) all the mathematics required for the first (indeed, in some ways, even more), (*b*) a profound understanding of the physical laws on which dynamics is based and (*c*) training and ability in the art and science of translating the physical conditions of an actual dynamical system into mathematical language. Moreover, since the second method involves continual reference to actual physical systems, it is especially difficult to present when the only available equipment is a blackboard and chalk.

Recognizing the foregoing facts, the author has built up a rather extensive "demonstration laboratory" consisting of various pieces and types of equipment which seemed most suitable as an aid in teaching this subject. Specific objectives in mind for the use of this equipment were: (1) to make clear the basic laws on which dynamics is founded; (2) to give physical meaning to the quantities and terminology employed in Lagrangian dynamics, such as, for example, generalized coordinates, degrees of freedom, degrees of constraint, kinetic energy in generalized coordinates, generalized forces; (3) to aid in giving physical meaning to various mathematical relations, particularly the Lagrangian equations of motion; (4) to acquaint the student with the wide variety of dynamical systems that may be encountered in actual practice; (5) to show some of these systems in action, as, for example, in the field of small oscillations; and finally (6) as a source of specific problems with which to give practice in that exceedingly important and sometimes difficult-to-grasp technique of translating the conditions of an actual dynamical system into mathematical language.

The demonstration laboratory consists of about 45 pieces—some academic in design and others very practical—which, except for a few special pieces, are mounted on supports attached to the walls or placed on narrow tables around the walls of a 20×25-ft lecture room. General views of most of the equipment are given in Figs. 1 to 6 inclusive. Certain pieces which seem to deserve a closer view are shown in Figs. 7 to 15.

Many of the photographs are self-explanatory. Hence we shall give only what description seems necessary in each case. Following this description, lists of pieces that are especially suitable for demonstrating various specific phases of dynamics will be given.

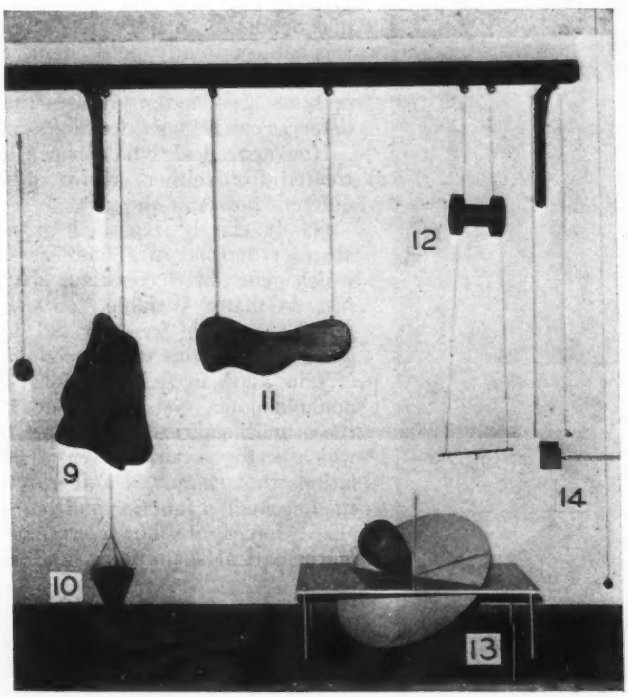

FIG. 2.

Description of Figures 1 to 6

Item *1* consists of a series of rods the ends of which are connected with strong cord.

Items *2* to *5* consist of lead balls connected with springs and are primarily for the purpose of demonstrating oscillatory motions.

Item *6* is an iron bar suspended from a spring; *7*, coupled pendulums; *8*, a gyroscope; *9*, a physical pendulum together with a simple pendulum having the same

period; *10*, a pendulum of variable mass (sand pours from a hole in the tip of the cone); *11*, a "rigid body" free to move in all directions under the action of gravity and springs; *12*, the well-known double pulley on a single shaft and wound with strings in such a way that it may move either up or down depending on the force applied to the lower strings; *13*, a model showing Euler angles (see Fig. 13 and corresponding description); *14*, a pendulum the support of which is free to move horizontally under the action of a spring; *15*, an arrangement for demonstrating the effect of "virtual mass" (see Fig. 14 and its description).

Item *16* is a double pendulum designed to show damping, proportional in one case to velocity and in another to relative velocity. The lower pendulum, consisting of a circular sector of sheet aluminum and (directly above it) an electromagnet, is suspended from a knife-edge bearing by parallel strips of iron. The upper pendulum, consisting of another aluminum sector and a mass, is supported by a short rod from a knife-edge bearing attached to the supporting strips of the first pendulum. The lower sector swings between the poles of an electromagnet attached to the base. The upper sector swings between the poles of the first mentioned magnet. The motion is damped by eddy currents set up in the sectors and, as can be seen, the damping force on the lower pendulum is proportional to its angular velocity while that on the upper is proportional to the relative angular velocity of the two.

Item *17* is a dumb-bell so mounted that the conservative forces due to the springs are, for most coordinates chosen, nonlinear.

Item *18* is a simple pendulum the length of which suddenly changes every half-cycle.

The dumb-bell *19* is supported from two strings tied to screw eyes placed a short distance apart on the periphery and at the center of the handle; if the lengths of the strings are properly chosen, this demonstrates beautifully the two modes of oscillation of the system.

Items *20* and *21* are merely mass-spring arrangements for vibration problems; *22*, a "rigid body" acted upon by various forces.

Item *23* consists of two containers suspended from springs, and presents a problem involving variable mass; sand pours from a hole in the tip of the cone and collects in the lower container, so that the upper mass decreases while the lower one increases.

Item *24* is a heavy rod suspended from several identical springs (see Fig. 11 and corresponding description); *25*, a box (simulating an automobile) suspended from four springs, one at each corner; *26*, a pendulum suspended from a V pulley which is free to run along a tightly stretched horizontal wire.

Item *27* is a double torsion pendulum consisting of two dumb-bells (small iron balls on brass rods) fastened to a tightly stretched piano wire 1 mm in diameter in a frame made of light angle iron welded together with a torch; such an arrangement makes an excellent demonstration of multiple torsional oscillations.

Item *28* is a horizontal bar (meter stick) supported at the middle from a knife-edge bearing and having a pendulum attached to either end.

Item *29* is a model for demonstrating the action of a watch when hung from a nail and free to swing in a vertical plane. The "watch" is a large piece of plywood, and the "balance wheel" is a solid iron disk (about 600 gm) mounted in hardened point bearings and fastened to a heavy clock spring in the manner of the hairspring of a watch. The disk must be started oscillating to show the action.

Item *30* consists of three disks mounted on the equivalent of a long flexible shaft, for showing torsional oscillations (see Fig. 8 and the accompanying description).

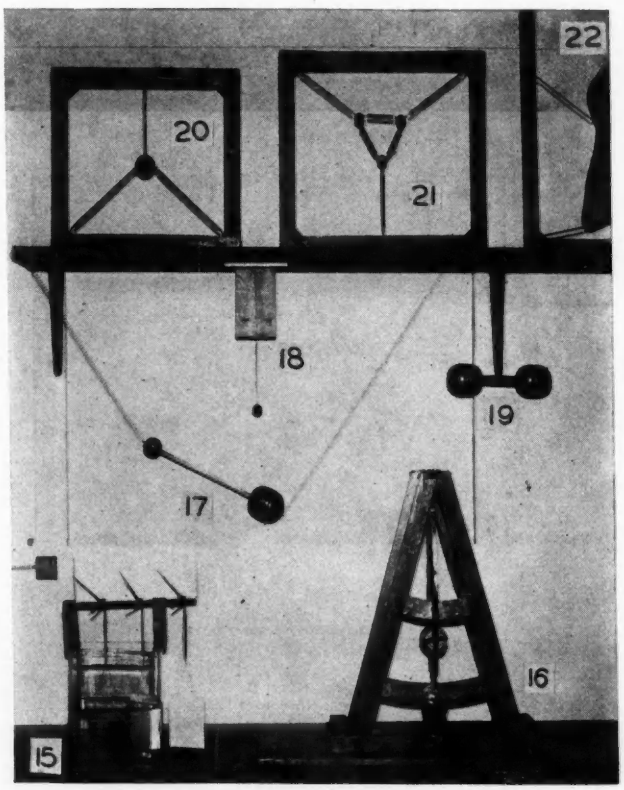

FIG. 3.

Item *31*, similar in construction to *27*, is used to show the effect of "virtual mass" by submerging it in water (see also Fig. 14 and accompanying description); *32* is a triple pendulum of unusual design (see Fig. 9 and description); *33*, merely a spring-mass-pulley arrangement.

Going from top to bottom, *34* consists of an iron ball, a short thin rod, a short length of flat steel spring (from a large clock spring), a short metal cylinder, another short length of steel spring and, finally, a solid cylinder which protrudes up from and forms a part of the heavy cast iron base. All parts are rigidly fastened together. The steel springs are held in deep slots by means of set screws. The short metal cylinder between springs actually consists of two cylinders held together by means of a tightly fitting male-female joint. Hence the upper spring can be rotated about a vertical axis and thereby set at any position relative to the lower one. The device gives an excellent demonstration of Lissajous figures for various settings of the springs.

When the pendulum in *35* swings, its length is continuously changed by the curved surfaces (thin sheet metal attached to wood).

Item *36* is a mass-spring arrangement used for the express purpose of demonstrating a system having many degrees of freedom (see Fig. 12 and its description); *37* and *38* are double pendulums.

Item *40* is an aid in discussing problems which involve the rotation of the earth. It is made of wire and a circular disk of sheet metal. Axes "at the center of the earth" are marked on the disk, and a rectangular system made of straight pieces of wire is soldered to the "surface of the earth."

Item *41* is merely a helix on which slides a heavy bead.

The demonstration equipment shown in Figs. 7 to 15 will now be described in some detail.

Fig. 7. Experimental determination of ellipses of inertia.—This apparatus is essentially a torsion pendulum. The irregularly shaped object (a piece of plywood), for which ellipses of inertia are to be obtained, is clamped in position between two parallel rectangular iron strips by means of a nut on an 8/32 screw which passes through a hole in the strips and the board. By loosening the nut, the board can be given any angular setting relative to the vertical strips. The ends of the strips are fastened with piano wire (1 mm in diameter) to the stout frame. The wire is kept taut by means of a screw at the top. The moment of inertia of the board about the supporting wire as an axis can be determined easily and accurately merely by timing, say, 100 oscillations with a stop watch and applying a well-known equation. This equation, of course, involves the torsional constant of the wire and the moment of inertia of the iron strips, which can be determined from a separate experiment. By changing the angular setting of the board in known steps of, say, 15°, sufficient data can be obtained in a very short time for plotting a complete ellipse of inertia about the point under consideration. Two such ellipses are shown on the board.

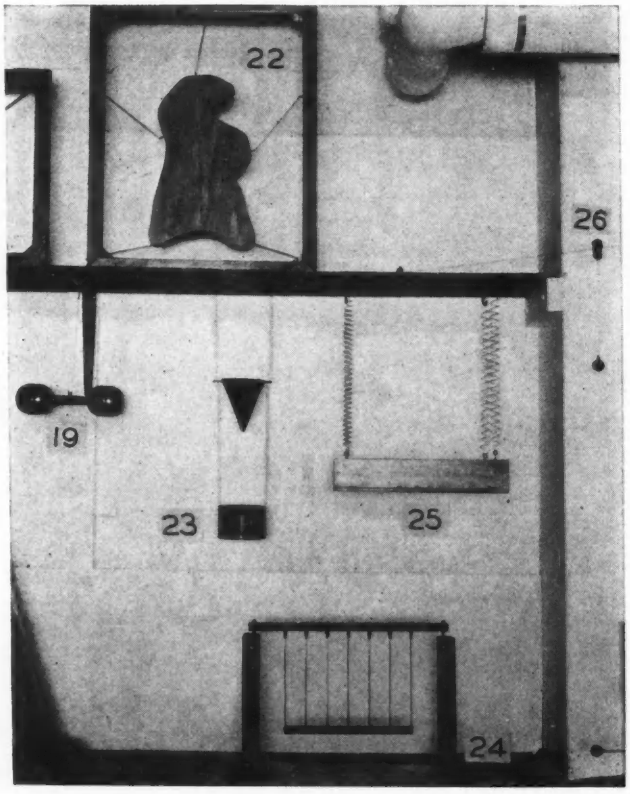

FIG. 4.

This demonstration never fails to make very real the meaning of the ellipse or ellipsoid of inertia. The apparatus could be modified to make it suitable for determining the entire ellipsoid of inertia about a point, but, owing to additional

complications involved, that is hardly worth while.

Fig. 8. Torsional vibrations in long shafts, etc.—Heavy disks made of wood faced with lead are mounted in ball bearings. To simulate long flexible shafting, the shafts of the individual disks are connected with short lengths of coil springs as shown. The torsional oscillations are best demonstrated by rotating (manually) the entire system and then momentarily stopping either or both of the smaller disks. If good bearings are used the rotation and oscillations will persist for some time.

Fig. 9. Triple pendulum of nonconventional design.—A rigid rod, to which the lowest bob is fastened, is supported at the top in widely separated point bearings. This type of support prevents sidewise motion. The remaining two bobs are suspended from strings which are tied to rigid arms extending out from the rod. Each arm is made by bending a strip of sheet metal around the rod and clamping it with 6/32 screws. Hence the position of either arm can easily be changed for any type of "tuning." The main support is made of water pipe, and the base is wood. In the early stages of dynamics this makes an excellent exercise in setting up expressions for kinetic and potential energies. It also provides a good problem in small oscillations.

Fig. 10. Triple torsional pendulum with forced motion.—The apparatus is intended primarily for demonstrating the three natural modes of oscillation of the system. An angle iron frame supports a tightly stretched piano wire to which three dumb-bells are rigidly fastened. Near the lower end of the wire is fastened a horizontal arm, about 4 in. long, which is held in contact with a "heart-shaped" cam. The cam is connected through a series of gears and pulleys to a variable-speed, direct-current motor (from an old Edison dictaphone) and is so shaped that, upon rotating, it imparts a sinoidal motion to the arm, which in turn imparts such a twist to the wire. If the frequency of rotation of the cam is the same as any one of the natural frequencies of the torsional system, that particular mode of vibration is violently excited. In designing the driving mechanism, the gear ratios and pulley ratios were so chosen that the rotational frequency of the cam could be made to correspond to any one of the natural frequencies of the system merely by shifting the belt connecting the motor shaft and larger pulleys, followed by a slight adjustment of the governor on the motor.

Fig. 11. Student project in computing small oscillations.—The apparatus is merely a heavy horizontal bar supported by a number of identical springs. The bar has six degrees of freedom and, when slightly displaced from the equilibrium position and released, oscillates with six frequencies. An outline of the results obtained by a student (Lt. W. L. Schwesinger) in computing the natural frequencies of the system is given on a chart attached to the base. One advantageous

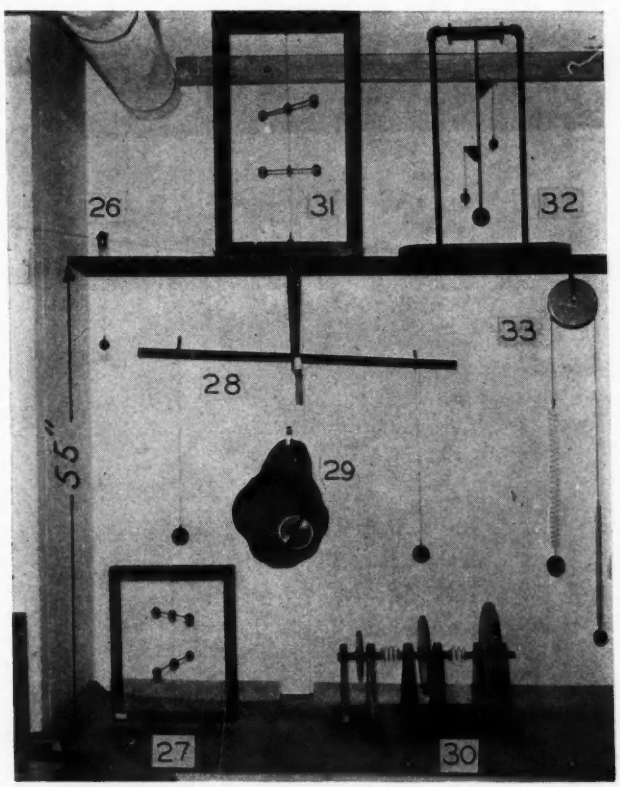

FIG. 5.

feature of this apparatus is that every frequency, except the rotational motion about the longitudinal axis of the bar, can be checked with a stop watch.

Fig. 12. Illustration of a system having many degrees of freedom.—This spring-mass arrangement serves well to illustrate the fact that a system of relatively simple appearance may require a large number of coordinates to represent its configuration. Even when we consider the balls as "particles" and assume that the bar does not rotate about its longitudinal axis, the system still has ten degrees of freedom. A discussion of this model never fails to impress students with the ideas of degrees of freedom and generalized coordinates.

Fig. 13. Euler angles.—The model, made of plywood, is for the purpose of making clear the Euler angles which are used so much in rigid body dynamics but are difficult to visualize from a blackboard drawing. The "top" shown in the figure may be regarded as any rigid body with one point fixed at the origin of coordinates.

Fig. 14. Qualitative demonstration of virtual mass.—Three lead plates of equal mass and dimensions are suspended from identical springs, two in water and one in air. Of the two in water, one is suspended so that it oscillates with its large area perpendicular to the direction of motion; the other, just as the one in air, so that its motion is edgewise. When the three are made to oscillate, the one in air has the shortest period, the one moving edgewise in the water a slightly longer period while the third one has a very noticeably longer period. This increase in period is due not to the damping of the water (except to a small extent) but to the effect of so-called *virtual mass*. Part of the work done in accelerating an object in a liquid goes to increasing the kinetic energy of the surrounding liquid, which, in effect, increases the mass of the object. The amount of increase, known as virtual mass, may be as great or even greater than the actual mass of an object. Hence virtual mass plays an important part in the motion of many dynamical systems. This arrangement demonstrates clearly the effect of virtual mass and also the fact that its value may depend very largely on the direction of motion.

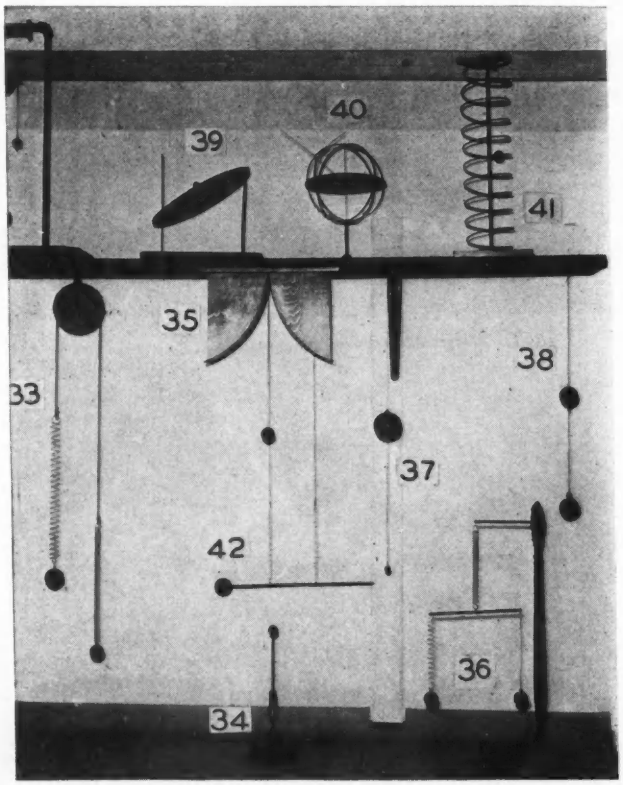

FIG. 6.

Fig. 15. Model for qualitative demonstrations of rotating axes, rotation of the earth, etc.—The model consists of a large circular disk of heavy plywood on which is mounted one octant of a sphere. This segment of the sphere (on which are mounted "axes fixed to the surface of the earth," a "long range gun," etc.) was made by covering a more-or-less spherical shell of screen wire with a $\frac{1}{4}$-in. layer of plaster of Paris and smoothing the soft plaster with a circular template. On the large disk there are mounted a Foucault pendulum and other systems involving rotating axes. The entire model is mounted on a vertical axis and can be rotated by a crank at the side (not shown).

We shall now list certain important general topics of dynamics

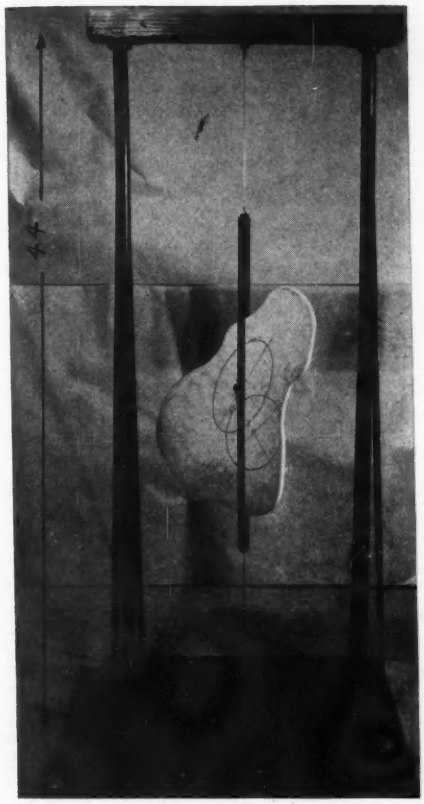

FIG. 7. Determination of ellipses of inertia.

and under each heading give, by figure and number, the pieces of apparatus best suited to demonstrate that particular phase of the subject.

Demonstrations Illustrating Background Material

Various topics which form an important part of the background necessary to a study of analytical dynamics are listed below together with a designation of the pieces of equipment with which they are best illustrated.

(1) Generalized coordinates and transformation equations: *33*, *37*, Fig. 6; *17*, *19*, Fig. 3; *14*, Fig. 2; *3*, *4*, *6*, *7*, Fig. 1; *32*, *30*, *27*, Fig. 5.

(2) Degrees of freedom: any of the models but especially *36*, Fig. 6. and those listed under (1) above.

(3) Degrees of constraint and equations of constraint: *37*, *41*, Fig. 6; any multiple pulley system illustrates this well.

(4) Velocity in generalized coordinates: *3*, *4*, *7*, Fig. 1; *14*, Fig. 2; *17*, *19*, Fig. 3; *27*, *29*, *30*, *33*, Fig. 5; *37*, *41*, Fig. 6.

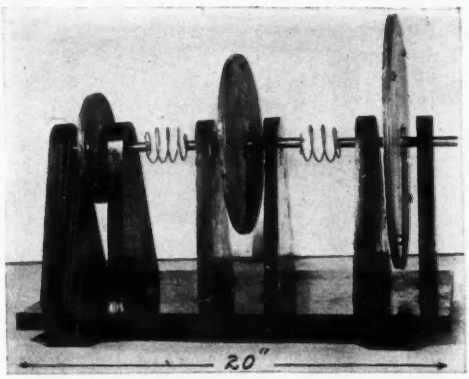

FIG. 8. Torsional vibrations in long shafts.

(5) Kinetic energy in generalized coordinates: same as listed under (4).

(6) Kinetic energy of a system with moving constraints and (or) moving coordinates: Fig. 15.

(7) Kinetic energy of rigid bodies: *8*, Fig. 1; Fig. 13; *22*, Fig. 4.

(8) Potential energy in generalized coordinates: *1* to *9*, Fig. 1; *11*, *12*, *14*, Fig. 2; *17* to *21*, Fig. 3; *22*, *25*, Fig. 4; *26* to *33*, Fig. 5; *34* to *38*, *41*, Fig. 6; Fig. 8; Fig. 9; Fig. 10; Fig. 11; Fig. 12.

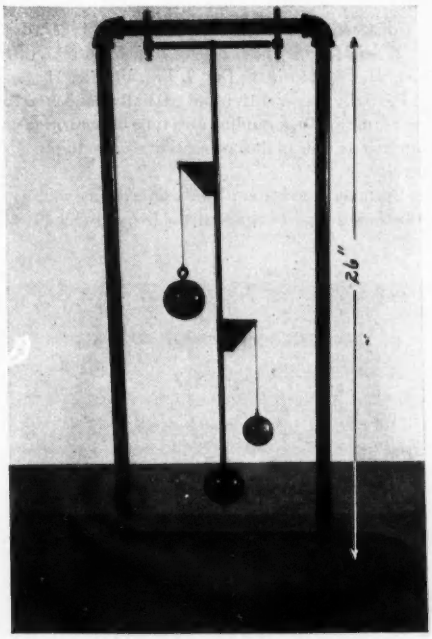

FIG. 9. A new triple pendulum.

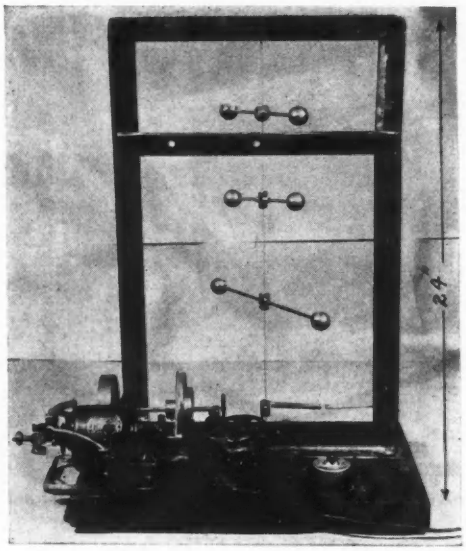

FIG. 10. Triple torsional pendulum with forced motion.

Demonstrations Illustrating Various Types of Dynamical Systems

(9) Conservative systems: *1* to *9*, Fig. 1; *11*, 12, 14, Fig. 2; *17* to *21*, Fig. 3; *24*, *25*, *26*, Fig. 4; *26* to *33*, Fig. 5; *34* to *38*, *41*, Fig. 6; Fig. 8; Fig. 9; Fig. 10; Fig. 11; Fig. 12.

(10) Partially conservative and partially nonconservative systems: same as (9), assuming any type of nonconservative forces acting as well as the conservative ones due to springs and gravity.

(11) Systems acted upon by dissipative as well as conservative forces: any of the systems listed under (9) where friction, viscous drag, and so forth, are assumed to be acting; also *16*, Fig. 3.

(12) Effect of the earth's rotation: *40*, Fig. 6; Fig. 15.

(13) Central force problems: *39*, Fig. 6; scaled drawings showing various types of orbit serve well for this; such drawings are included in this laboratory, but are not shown here.

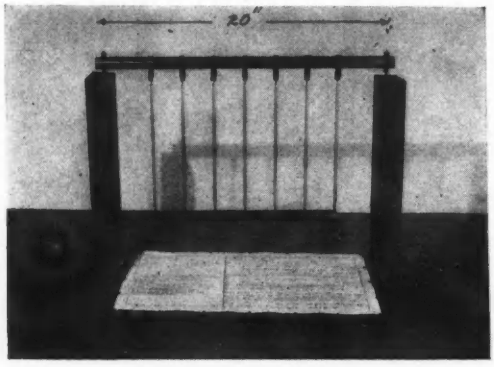

FIG. 11. Project in computing small oscillations.

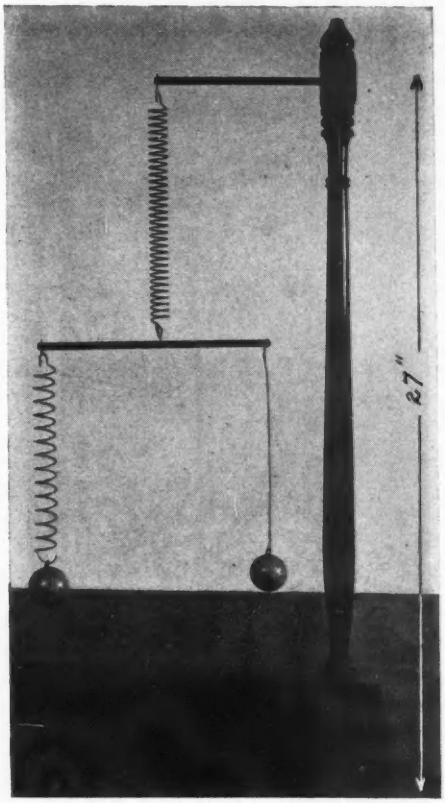

FIG. 12. System with many degrees of freedom.

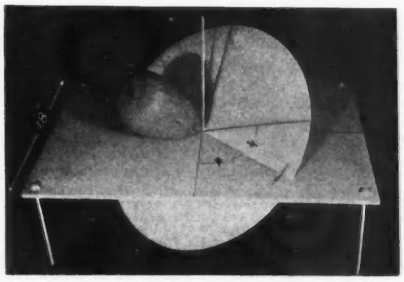

FIG. 13. Euler angles.

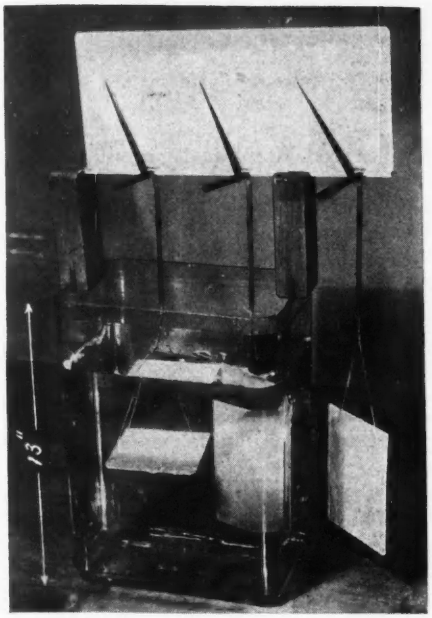

FIG. 14. Demonstration of virtual mass.

(14) Oscillation problems: many of the models can be used to illustrate oscillatory motion, but especially useful are those in Fig. 1; *14*, Fig. 2; *19*, *20*, *21*, Fig. 3; *24*, *25*, Fig. 4; *28* to *33*, Fig. 5; *34*, *35*, *37*, Fig. 6.

(15) Rigid body dynamics: *8*, Fig. 1; *11*, Fig. 2; *22*, Fig. 4; *29*, Fig. 5; Fig. 7; Fig. 13.

(16) Systems with variable mass: *10*, Fig. 2; *23*, Fig. 4.

(17) Virtual mass: *31*, Fig. 5 (periods when immersed in water greatly increased); Fig. 14.

FIG. 15. Model for qualitative demonstration of rotating systems.

General Remarks

It is the author's belief that the results obtained from this equipment have far more than justified the time and effort spent in its design and construction. It is contemplated that additions will be made from time to time.

Most of the apparatus was constructed by Mr. Charles Asawa. For his patience and more than ordinary skill at this work, the author is greatly indebted to him.

National Research Council Predoctoral Fellowships

THE National Research Council has received $335,000 from the Rockefeller Foundation for the establishment of a temporary program of predoctoral fellowships in the sciences. The fellowships will enable selected young men whose education has been interrupted by war work to devote essentially full time to the completion of their work for the doctorate. Although the program probably cannot be inaugurated immediately, this announcement is made so that students now in war work will realize that perseverance in that important task will not prejudice their opportunity to pursue formal education later, and that it may be inadvisable to make commitments at present for postwar employment.

Obviously, this grant, while generous, is not sufficient to furnish financial aid to more than a moderate fraction of those who should resume graduate work in the sciences. Therefore, other types of assistance heretofore available should not be curtailed. The plan is not to divert too many qualified candidates from part-time teaching positions, since it is expected that colleges and universities will, after the war, be overburdened with undergraduate students in the sciences.

The National Research Council will continue to administer its post-doctoral fellowship program, which has been an outstanding factor in building up the scientific competence of the country.

A Classical Experiment Illustrating the Notion of "Jerk"

P. LeCorbeiller
Cruft Laboratory, Harvard University, Cambridge, Massachusetts

1. It seems customary, both in this country[1,2,3] and abroad,[4,5] to introduce the notion of inertia by presenting the experiment of a weight hanging from a thread with an extra piece of thread hanging below the weight (Fig. 1). The experiment is described and explained more or less as follows:

> A steady downward pull on the lower thread will break the upper one, because it must sustain the heavy weight in addition to the pull; a quick downward snap will usually break the lower thread only, the upper one being momentarily protected from much stretching by the inertia of the weight.[1]

In trying to translate this qualitative reasoning into mathematical language, one finds that everything depends upon the precise definition of the applied force f as a function of time t. In particular, a force constant in time could never break the lower thread; while a force equal to zero for t negative, and constant but finite for t positive, would never break the upper one. Since the derivative of f, for $t=0$, is zero in the first case and infinite in the second, one is led to think that the time-rate of change of the applied force may be the controlling factor in the contrasting effects observed. Accordingly, it will be assumed in what follows that the applied force is zero for t negative and increases proportionately to time for t positive. It will then be found that when the derivative of $f(t)$ is less than a certain value, the upper thread breaks, and when it exceeds this value, the lower thread breaks. More complicated assumptions about $f(t)$ would give different and less simple results.

2. The two pieces of thread will be assumed to be equivalent to two identical springs of spring-constant k and of negligible mass compared with the mass m of the weight (Fig. 2). It will be assumed that each spring or thread will break under a force $2mg$. Let x_1 be the difference between the actual length of the lower, or No. 1, thread at any time t and its length when not strained; let x_2 be the difference between the length of the upper, or No. 2, thread at time t and its length when it is not supporting any weight. Let f_1, f_2 be the downward forces acting on the lower and upper threads. We have

$$f_1 = kx_1. \tag{1}$$

This force is zero before we begin to pull, and for t positive we shall assume that its expression is

$$f_1 = 2mg(t/t_1), \tag{2}$$

as shown by the straight lines in Figs. 3 and 4. A

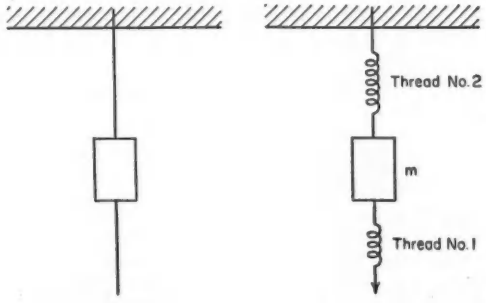

FIG. 1. Classical experiment of weight and threads.

FIG. 2. Problem in mechanics.

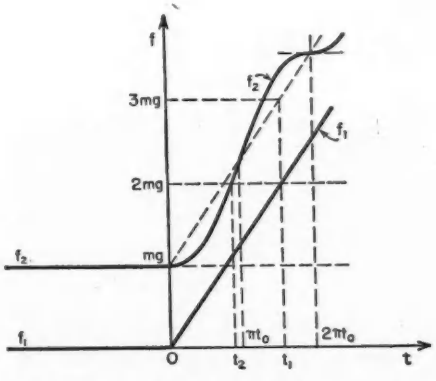

FIG. 3. Slowly increasing pull f_1 on lower thread; upper thread snaps first.

[1] Saunders, *A survey of physics for college students* (ed. 3), p. 47.
[2] Black and Davis, *Practical physics*, p. 147.
[3] Kimball, *A college textbook of physics*, p. 20.
[4] Muller-Pouillet, *Lehrbuch der Physik* (ed. 11), vol. I, p. 319.
[5] Pohl, *Mechanik and Akustik*, p. 23.

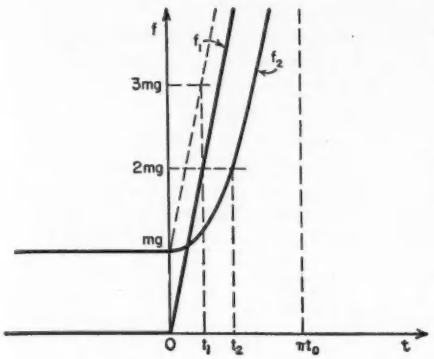

FIG. 4. Rapidly increasing pull f_1 on lower thread ("jerk"); lower thread snaps first.

large value of the time-constant t_1 will correspond to a "steady downward pull," and a *small* value of t_1 to a "quick downward snap." The lower thread alone would break at time $t=t_1$.

For the upper thread we have the two equations

$$f_2=kx_2, \tag{3}$$

$$f_2=f_1+mg-m(\mathrm{d}^2x_2/\mathrm{d}t^2); \tag{4}$$

hence

$$(m/k)(\mathrm{d}^2f_2/\mathrm{d}t^2)+f_2=f_1+mg. \tag{5}$$

For $t<0$ we have $f_2=mg$, and for $t>0$ we have

$$(m/k)(\mathrm{d}^2f_2/\mathrm{d}t^2)+f_2=(2mg/t_1)t+mg. \tag{6}$$

3. The general solution of Eq. (6) is

$$f_2=A\sin((k/m)^{\frac{1}{2}}t+\varphi)+(2mg/t_1)t+mg. \tag{7}$$

We determine the integration constants A and φ by the initial conditions $f_2=mg$, $\mathrm{d}f_2/\mathrm{d}t=0$ for $t=0$. Then Eq. (7) becomes

$$f_2=-\frac{2mg}{t_1}\left(\frac{m}{k}\right)^{\frac{1}{2}}\sin\left(\frac{k}{m}\right)^{\frac{1}{2}}t+\frac{2mg}{t_1}t+mg. \tag{8}$$

Let us put

$$t_0=(m/k)^{\frac{1}{2}}. \tag{9}$$

This time-constant t_0 is independent of the applied force; in fact, $2\pi t_0$ is the period of free oscillation of the mass suspended to the elastic upper thread. Equation (8) becomes

$$f_2/2mg=[-t_0/t_1][\sin(t/t_0)]+t/t_1+\tfrac{1}{2}. \tag{10}$$

The variations of f_2 are shown by the upper curve in Fig. 3. Assuming that the lower thread has not broken, the upper thread will break when $f_2=2mg$ and therefore at a time t_2 given by the equation

$$[-t_0/t_1][\sin(t_2/t_0)]+t_2/t_1=\tfrac{1}{2}, \tag{11}$$

which, from Fig. 3, can have only one root.

Setting $t_2=t_1=t'$ in Eq. (11), we see that the two times t_1 and t_2 are equal when

$$2\sin(t'/t_0)=t'/t_0. \tag{12}$$

This equation yields only one solution,

$$t'=1.895t_0=1.895(m/k)^{\frac{1}{2}}.$$

If the applied force reaches the breaking strength $2mg$ at the time t', both threads are simultaneously stressed to the breaking point and they break simultaneously at t'. If the force on the upper, or No. 2, thread reaches the breaking strength $2mg$ while the force on the lower, or No. 1, thread has not reached this value ($t_2<t_1$), the upper thread breaks first, and the experiment ends. If, on the contrary, the force on the lower thread reaches $2mg$ while the force on the upper thread has not reached this value ($t_1<t_2$), the lower thread breaks first.

4. The discussion does not end here. Assume that $t_1<1.895t_0$, so that the lower thread breaks first. The weight now hangs from the upper thread and oscillates freely, the initial position and initial downward velocity being known; if the amplitude of this oscillation exceeds mg/k, the upper thread breaks when the extension exceeds $2mg/k$. Putting $\theta=t_1/t_0$, one finds that the critical value of θ is given by the equation

$$((1-\cos\theta)/\theta)^2+(1-(\sin\theta/\theta))^2=\tfrac{1}{4},$$

which has one root, $\theta=1.03$.

Let $S[\equiv \mathrm{d}f_1/\mathrm{d}t=2mg/t_1]$ be the rate of increase of the applied force, let $F_{br}[\equiv 2mg]$ be the force under which either thread breaks, and let $T[\equiv 2\pi(m/k)^{\frac{1}{2}}]$ be the period of oscillation of the weight hanging from thread No. 2. Then, if $S<3.31(F_{br}/T)$, the upper thread will break; if $3.31(F_{br}/T)<S<6.10(F_{br}/T)$, the lower thread will break first and the upper one next; if $6.10(F_{br}/T)<S$, the lower thread only will break.

5. The rate of increase of acceleration is a quantity that must be kept small for the comfort of the passengers riding in an elevator, a subway train or a car; if too great, it might even be the cause of accidents. In planning a subway, for instance, the designer would like to use very large accelerations and decelerations, in order to reduce the time of travel between stops. During constant acceleration, a passenger maintains his balance by holding on to a support of some kind and in this

way may accommodate himself to high accelerations. But if, starting from rest, a high acceleration is reached in a very short time, the traveller receives a "jerk" which may make him lose his grip and fall. In an elevator the internal feelings of discomfort due to a sudden change of acceleration are much worse than those due to a constant acceleration. Similar considerations enter into the design of automobile shock-absorbers.[6]

In conclusion, we have found that the verbal explanation given of the "weight and threads" experiment is essentially correct, but that the distinction between "a steady downward pull" and "a quick downward snap" involves the time derivative of the applied force. It may be doubted whether the experiment can be understood by beginners to whom even the notion of inertia is none too clear. On the other hand, it may well serve to demonstrate to more advanced students the effects of a variable force, a subject of growing technical importance.

The author is indebted to Professor P. W. Bridgman and to Dr. A. H. Wing, Jr. for helpful suggestions.

[6] Den Hartog, *Mechanical vibrations*, p. 145.

What is the "Geiger-Nuttall Law"?*

J. G. BECKERLEY
Columbia University, New York, New York

WHEN experimental measurements indicate a relationship between quantities for which no theoretical basis exists, it is usual to relate these quantities by an equation. Such a relationship is called an *empirical equation*.

The empirical equation must be considered a strictly temporary device, since it is no longer useful when a consistent theory or hypothesis is proposed. In fact, it is *not* necessary that the proposed theoretical relation fit the experimental data better than the empirical formula does. The approximations involved in the theoretical derivation or even the reliability of the data may make the experimental points deviate considerably from the theoretical curve.

From the point of view of an instructor it is desirable to distinguish carefully between those relationships that can be derived from a logical theory and those that are purely empirical. Seldom should an empirical formula be elevated to a prominent position, and this is particularly true when a satisfactory theoretical equation exists.

One reason for "soft-pedaling" empirical relations is their lack of uniqueness: often several simple formulas will fit equally well a given set of measurements.[1] This occurs in the description of alpha-particle emission by naturally radioactive nuclei. The empirical formula relating the disintegration constant λ (the fraction of radioactive nuclei disintegrating per unit time) and the alpha-particle range R is called the "Geiger-Nuttall law."

In describing this "law," current books on nuclear physics have introduced at least *four* versions.[2] The references all agree that one variable in the "law" is $\log_{10}\lambda$, the logarithm of the disintegration constant, but disagree on the second variable. The following empirical equations are used in the various books:

$$\begin{aligned} &\log\lambda = a\log R + b, && [\text{Ref. } 2(a), (b)] \\ &\log\lambda = a'E + b', && [\text{Ref. } 2(c)] \\ &\log\lambda = a''\log E + b'', && [\text{Ref. } 2(d)] \\ &\log\lambda = a'''(1/v) + b''' && [\text{Ref. } 2(e)] \end{aligned} \qquad (1)$$

where R is the alpha-particle range, E the disintegration energy, and v the initial velocity of the alpha-particle. The a's and b's are constants chosen to secure the best "fit;" the a's are constant for all three radioactive series, whereas

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[1] An example is the classical and quantum-mechanical derivations of Richardson's equation, both derived equations fitting the experimental data, at least in the usual range of values, despite differences in form. In such a case additional experiments are necessary to eliminate the ambiguity.

[2] (*a*) N. Feather, *An introduction to nuclear physics* (Cambridge Univ. Press, 1936), p. 39; (*b*) F. Rasetti, *Elements of nuclear physics* (Prentice-Hall, 1936), p. 99; (*c*) G. Gamow, *Structure of atomic nuclei and nuclear transformations* (Oxford, 1937), p. 86; (*d*) E. Pollard and W. Davidson, *Applied nuclear physics* (Wiley, 1942). p. 120; (*e*) H. Bethe, Rev. Mod. Phys. 9, 164 (1937).

different values of the b's must be chosen for each of the series. Each of Eqs. (1) fits the experimental data with reasonable accuracy. Yet *historically*[3] only the first is justified; that is, if the relation is labelled "The Geiger-Nuttall law," then the variables are $\log \lambda$ and $\log R$.

The multiplicity of possible simple empirical relations is due to the fact that the velocities, energies or ranges are confined to the approximate intervals:[4]

$$\begin{aligned} &1.41\times10^9 \text{ cm/sec} \leq v \leq 1.94\times10^9 \text{ cm/sec},\\ &4.15\times10^6 \text{ ev} \leq E \leq 7.83\times10^6 \text{ ev},\\ &2.64 \text{ cm} \leq R \leq 7.10 \text{ cm}. \end{aligned} \tag{2}$$

If the variation of the velocity is considered, then the change of the velocity from its mean (1.67×10^9 cm/sec) is about 16 percent. It is obvious that since v varies over such a small interval, the variables v^2 (or E) and v^{-1} might be chosen as well as v. This was recognized in 1912 by Swinne[5] who showed that Geiger and Nuttall's data could be represented empirically by $\log \lambda = a + bv^n$, with $n=1$ or 2. (He would have been 16 years ahead of time had he proposed $n=-1$!) Of course, if v^2 is a suitable variable, then $\log v^2$ (or $\log E$) would be valid as well as $\log v^3$, or $\log R$, since R is proportional[6] to v^3 in the range of values of Eqs. (2). The validity of these variables is justified by the equations:

$$\begin{aligned} &\frac{\Delta(1/v)}{1/v} = -\frac{\Delta v}{v}, \quad \frac{\Delta E}{E} = 2\frac{\Delta v}{v},\\ &\Delta \log (E/\bar{E}) = 2\Delta v/v,\\ &\Delta \log (R/\bar{R}) = 3\Delta v/v, \end{aligned} \tag{3}$$

where $\bar{E}$, $\bar{R}$ are the mean values of E and R. These equations follow from $E=\frac{1}{2}Mv^2$ and $R=Cv^3$, if $\Delta v/v \ll 1$. Thus all the fractional deviations are of the order of $\Delta v/v$. Hence if $\Delta v/v$ were zero, then $\Delta E/E$, $\cdots$, would be zero likewise. In the case at hand, $\Delta v/v \simeq \pm 0.16$, so that if v is a suitable variable, then all the other variables are not equally suitable. If $\log R$ or $\log E$ is plotted as a function of $\log \lambda$ by using experimental values of R, E, λ, a set of points lying approximately on a straight line is obtained. A plot of E *versus* $\log \lambda$ yields a line of definite curvature.[7]

Actually the estimate $\Delta v/v \simeq \pm 0.16$ is pessimistic; for, if the apparently anomalous C' disintegrations are excluded, we obtain $\Delta v/v \simeq \pm 0.12$, ± 0.095, ± 0.094 for the Th, U-Ra and Ac series. Consequently, any of the empirical Eqs. (1) can be made to fit the experimental data fairly well.

In fact, if $\Delta v/v$ is assumed small it is easy to derive other empirical formulas to fit the data. For example, even $\log \lambda = A \sin(v/\bar{v}) + B$, where $\bar{v}$ is the mean value of the velocity, can be made to fit quite well, although for obvious reasons a periodic variable would be difficult to justify theoretically.

CONCLUSION

Since in the process of curve fitting, a variety of simple functions of v, E or R are suitable variables for expressing the empirical relation between $\log \lambda$ and any one of these functions, it is clear that the discrepancies in the references do not constitute a serious error.

Nevertheless, for the sake of consistency, any book on nuclear physics should state:

(1) That, *historically*, the Geiger-Nuttall law is an *empirical* equation, $\log \lambda = a \log R + b$, with a a constant for all natural radioactive series and b a constant for each of the three radioactive series.

(2) That, by representing alpha-particle decay as a quantum-mechanical process in which an alpha-particle penetrates a suitable potential barrier, one can derive the theoretical relation,[8] $\ln \lambda = -(a'/v) + b'$, where a' is $4\pi Ze^2/h$ and b' is a slowly varying function of the product-nucleus' radius. This relation is the theoretically justified analogue to the empirical Geiger-Nuttall law and should be given considerably more emphasis than the empirical law.

(3) That, in order to compare the theoretical formula with the experimental data, it is more convenient to calculate b' from the experimental values, and from b' to deduce the nuclear radii of the radioactive elements.[9] This process is much more sensitive in revealing deficiencies of the theory than is the usual curve-fitting method. As has been shown here, the latter process does not even assure one that the proper variables are involved in the formula for $\log \lambda$.

[3] H. Geiger and J. M. Nuttall, Phil. Mag. **22**, 613 (1911).
[4] For energies, see ref. 2(*e*), p. 166, table XXIX; for conversion to ranges, see M. Livingston and H. Bethe, Rev. Mod. Phys. **9**, 266 (1937).
[5] R. Swinne, Physik. Zeits. **13**, 14 (1912).
[6] M. Livingston and H. Bethe, ref. 4.
[7] See ref. 2(*c*), p. 86, Fig. 22.
[8] This is not the exact formula [see ref. 2(*e*), Eqs. (587) and (589)], but it is certainly as valid as the simplified nuclear model itself. In the expression for a', Z is the atomic number of the product nucleus.
[9] See ref. 2(*e*), pp. 164–168.

To the Master's Degree in Four Years

ALAN HAZELTINE*
Maplewood, New Jersey

A CURRICULUM is proposed that will lead to a bachelor's degree in three years and will provide an adequate preparation for graduate study in a fourth year leading to a master's degree in any of the broader fields of science or engineering.[1] Among such broader fields are included physics, mathematics, general engineering, mechanical engineering and electrical engineering. For relatively specialized fields, such as chemistry and chemical engineering, the proposed curriculum would provide a strong foundation on which could be built a more extended course of graduate study in the specialty. Moreover, as has been argued elsewhere,[2] the proposed type of curriculum is particularly suited for a *cultural* course, designed to give the fullest understanding of the modern world—the age of physical science and its applications. Thus the immature student embarking on such a course need not decide on his port of destiny until he has had three years to view the fields of knowledge and to appraise his tastes and capabilities.

We owe it to our students to provide their needed education in the least possible time, so as to give them an early start on their careers. There has been a long-time tendency to delay the age of graduation. I recently looked up the ages of graduation from college of a dozen well-known Americans chosen at random: they showed a progressive increase from 15 for Cotton Mather, Harvard 1678, to 22 for Franklin D. Roosevelt, Harvard 1904. In no case was the age of graduation beyond 22, which is about the present *average*. On top of this are the years required for graduate courses—now more commonly taken than a generation ago, especially by engineering students. For comparison with conditions abroad, I might quote from a recent report:[3]

> Before the war in Germany the average age for receiving a doctor's degree was twenty-four years. In the United States it was thirty years—an age when some of the most creative years and the prime of life, physically, are already past.

The reason for this deferment of graduation is the failure of teaching methods to keep pace with the advance in factual knowledge. This is not altogether necessary: I believe that *we can now so improve our methods of presentation of physical science and engineering as to save the student one college year*. Such improvements lie in two directions: first, in the revision of individual courses; and second, in the better coordination of different courses having overlapping subject matter.

Perhaps no course needs revision as badly as the physics course. We know the great generalizations in physics developed during the nineteenth century, but we do not fully incorporate them in our teaching. We introduce mechanical forces with Archimedes' laws of equilibrium and Newton's third law of motion, without recognizing that these are consequences of the conservation of energy. We reckon temperature on some arbitrary scale, although the only rational scale depends on the second law of thermodynamics. We introduce electricity and magnetism through Coulomb's laws for point charges and poles, without recognizing that these laws are consequences of Faraday's ideas of dielectric and magnetic fluxes, together with the conservation of energy. We describe light as the fictitious vibration of a nonexistent medium, although we know that it is an electromagnetic wave. In short, we present basic physics as a series of disconnected subjects, in disregard of the great unifying principles. This is not good education—and it wastes valuable time.

It has been customary to organize a curriculum by assigning a certain number of semester-hours to each teaching department, or to each subject within a department, and then to leave the arrangement of the course entirely to that department, or to an individual teacher. The result has been duplication, gaps and diversities in treatment, all lowering the efficiency of instruction. *A curriculum should be designed as a whole*, with full correlation of its parts, just as is done in designing a machine. No department should disparage the work of another nor attempt to

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* Lately Chairman of Department of Physics, Stevens Institute of Technology.

[1] Such a proposal was recently made to the Trustees of Stevens Institute of Technology and is now under consideration by the faculty.

[2] A. Hazeltine, "Some revisions of engineering education," J. Eng. Ed. **35**, 233 (1944).

[3] S. L. Pressey, Science **100**, 10 (Sept. 22, 1944 supplement).

repeat it; whatever reviews are needed should come in incidentally, as parts of advanced topics. Often the physics teacher and the engineering teacher in the same field talk different languages. Thus, in electrostatics, "the capacity of a condenser" in the physics course becomes "the capacitance of a capacitor" in the electrical engineering course; and, worse still, the physicist looks only at its surface charges, while the engineer looks at its dielectric flux. Naturally, the student going from the physics to the engineering department can hardly recognize the same subject. Conferences between departments should correct such conditions, provided that the physics teacher is willing to learn from his engineering colleague, who is a specialist in his field. The engineering teacher *has* to know a good deal of physics; the physics teacher *ought* to know a good deal of engineering.

The improvements in teaching will naturally result in more concentrated and more rigorous courses. To be able to profit from such courses, students will need to enter college with adequate preparation, such as is now provided by the stronger high schools of the country. The entering student should have had training in algebra, plane and solid geometry, trigonometry, physics, chemistry and (preferably) French or German, besides the usual work in English and history. The really bright high-school graduate who has missed some of these subjects could remove his deficiencies by a few weeks' tutoring in the summer before entrance—which might well be arranged by the college.

The college year can advantageously be lengthened: those colleges who draw their students mainly from the stronger high schools might find sufficient time in two semesters of 17 weeks each, or three trimesters averaging 12 weeks each, inclusive of examination time but exclusive of holidays; colleges less fortunately situated might have to adopt a slower pace, with three 13-week trimesters, extending from the second week in September to July 4. Even in the latter case, the student would get 10-week summer vacations, besides a week between consecutive trimesters, a Thanksgiving recess and the national holidays; these should provide sufficient periods for relaxation and for make-up work. The total time available in three such college years would be from 80 to over 90 percent of that now used in four years, each of two 16-week terms. The improvements in teaching and in reorganization should fully make up for these slight decreases in working time. I believe that the possible gains will be found to be much greater; so that the course as a whole will actually be strengthened.

The questions may be asked whether the preceding arguments do not apply also to liberal arts curriculums. The answer is *no:* the curriculum with which we are concerned is distinguished by its *concentration;* its predominant subjects, physical science and engineering, are taught from specially prepared texts, with every page presenting new ideas—and precise ones. Courses in which humanistic subjects predominate are inherently discursive; a great volume of reading of original sources is needed in order to sift out a few ideas—and these are not precise. Furthermore, a curriculum having many electives cannot be designed as a whole. Thus the liberal arts course will always need more time than the engineering course for a comparable accomplishment.

There are other places for four-year undergraduate curriculums. Students ready to specialize on entering college may need the time to study their specialty concurrently with a general education. Also students desiring a general training in science or engineering, but not having the preparation demanded for the three-year curriculum, or not having the mental equipment and stamina to keep up the faster pace, will continue to demand the longer curriculum.

The desirability of engineering training for the industrial physicist has been recognized.[4] Its desirability for the future physics teacher is fully as great: his classes will be largely made up of engineering students, and his courses will receive their deserved attention only if they carry the engineering appeal. Moreover, for his illustrations of quantitative physical laws he cannot go to the forest; he can find them only in the apparatus designed by the engineer. The undergraduate curriculum suggested in Table I is essentially one in general engineering, with strong courses in the basic sciences and with adequate attention to humanistic-social studies, as recently recommended.[5] In addition to the

[4] H. E. Ives, "Some aspects of research in applied physics," J. Eng. Ed. **22**, 82 (1931); H. L. Dodge, "Training physicists for industry," Am. J. Phys. (Am. Phys. T.) **4**, 167 (1936); AAPT Committee on the Training of Physicists for Industry, Am. J. Phys. (Am. Phys. T.) **6**, 83 (1938); R. Gunn, "The research physicist—his characteristics and training," Am. J. Phys. **10**, 181 (1942).

[5] Report of SPEE Committee on Engineering Education After the War, J. Eng. Ed. **34**, 595 (1944). This authoritative study sustains many of the arguments in the present paper.

TABLE I. Undergraduate curriculum, hours per week in classroom and laboratory.

First year		Second year		Third year	
Humanities	5+ 0	Humanities	3+ 2	Humanities elective	3+ 3
Mathematics	4+ 0	Mathematics	3+ 2	Economics	3+ 0
Physics	4+ 3	Physics	3+ 3	Electrical engineering	3+ 3
Mechanism	3+ 6	Mechanics	3+ 3	Mechanical engineering	4+ 3
Shop practice	0+ 4	Chemistry	4+ 3	Chemistry	3+ 3
Physical education	0+ 3	Physical education	0+ 3	Physical education	0+ 3
	16+16		16+16		16+15

tabulated courses, a six-weeks course in surveying is suggested in the summer following the first year; if held at a camp, it affords outdoor recreation and the opportunity for all members of the class to know one another, fostering class spirit.

The time scheduled for "laboratory" (including drawing, shop, conference and computation periods) is greater than is now usual, the increase being largely intended for supervised computations—a very effective form of instruction. On this account, less time is needed for outside preparation, which is expected to take *the same* number of hours as are scheduled for the classroom—not *twice* as many, as is now usual. The total working week (classroom+preparation +laboratory) is thus 48 or 47 hours.

The primary objective of all "laboratory" courses is to give an understanding of basic principles by concrete applications; manipulative drills are by-products. So all of these courses should be closely correlated with the corresponding classroom courses. Often it is desirable to carry a topic further in the laboratory than has been done in the classroom; and it is then helpful to give a short lecture at the beginning of each three-hour laboratory period. In physics, mechanics and the two engineering courses, the laboratory time may well be divided equally between experimental work and supervised computations, to be performed on alternate weeks; so that laboratory apparatus need be sufficient for only half of the class.

Some discussion of the individual courses will now be given, with emphasis on the proposed departures from present usual practice.

Humanities. A minor part of the scheduled time is for oral and written English expression. The major part is for history and literature, preferably combined into a single course by the reading of original historical sources or their best available translations (as has been done at Stevens in recent years). A limited number of options may be provided, especially in the latter part of the course. The humanities elective in the third year should be chosen to suit the student's subsequent specialty: for those who will take a graduate physics course, the history of mathematical and physical ideas would be appropriate—unless the time is needed for French or German by students who have taken neither in high school. The "laboratory" time is intended for library reference and report writing on the chosen elective; for experience shows that the teaching of English composition is effective only when the subject matter is of real interest to the student.

Economics, the remaining humanistic study, gives the student some understanding of the organization of modern society—particularly in business and industry, with which the ordinary citizen is most intimately concerned. As in science and engineering, a full understanding calls for quantitative expression and thus for some training in accounting.

Mathematics. After an introductory few weeks devoted to a logical presentation of the elementary algebraic processes, serving incidentally as a review, calculus should be started, since the concepts of derivative and integral will occur early in the concurrent physics course. Analytic geometry should not be given as a distinct subject: much of the conventional course is both dull and insignificant; and the remainder is better incorporated with calculus or with algebraic topics—such as *nomographs*, which give excellent applications of the equation of the straight line. The first year should emphasize the *reality* of mathematics; the second year is the time for the *formal* processes, such as the use of complex numbers in integration. The last semester or trimester should be devoted to differential equations, with emphasis on numerical solutions and on linear differential equations with constant coefficients, as exemplified by dynamical systems and electric circuits of a finite number of degrees of freedom. Students should become familiar with the use of complex quantities for steady-state sinusoidal solutions

(vibrations and alternating currents) and with the Heaviside operational calculus for transients. In fact, the entire course might well be built around cleverly selected problems of physics and engineering; and many such problems may advantageously be treated in the mathematics course alone. Some of the longer problems of the second year can be worked out in the two-hour period provided for supervised computations.

Physics. The first major division of the course would be the *mechanics of solids*, including the following topics: geometry, logically developed from facts of observation (which are the proper axioms) and applied to mechanisms; kinematics, applied to the operation of such mechanisms; dynamics, built on the general observation of conservation of energy and using this to derive the laws of forces and torques, which are derivatives of energy with respect to displacement; kinetics, dealing with mass and accelerations; elasticity, including the compliance of springs and the strength of materials. The next division would be the *mechanics of fluids*, including: dynamics, with the hydrostatic laws derived from pressure as the derivative of energy with respect to volumetric displacement; kinetics, including Bernoulli's principle; viscosity; surface tension; mechanics of soils, applying hydrostatics and viscosity and introducing *fields of flow* through tangible quantities, for subsequent extensions to heat conduction and electricity; elasticity of fluids, particularly isothermal and adiabatic expansion and a purely mechanical definition of an ideal gas, with temperature entering only as a *physical quality*, having an arbitrary scale. *Heat* would start with the second law of thermodynamics and the Carnot cycle, leading to a physical definition of entropy change as *that part of an energy transfer which is unavailable at a chosen standard temperature;* temperature would then be defined as a *physical quantity*, the derivative of energy with respect to entropy; appropriate Carnot cycles would be employed to derive the ideal gas law, $pV = MRT$, the formulas for the ratio and the difference of the specific heats of any substance in terms of the bulk moduli and the coefficient of expansion, and Clapeyron's equation for change in phase—all exhibiting the power of thermodynamic reasoning. The first year should end with *vibrations and waves*, which apply kinetics and elasticity of solids and fluids: with free and forced sinusoidal vibrations should be introduced the concept of mechanical impedance and the use of vectors to represent it and the sinusoidal forces and velocities; wave pulses in rods or pipes (water hammer) serve well in deriving (without calculus) the formulas for velocity of propagation and natural impedance and for reflections at discontinuities, with ready generalization to waves of any form, including sound waves, as in organ pipes; plane waves in space, their reflection and refraction are similarly treated; radiation from a point source requires only elementary calculus;[6] the peculiarities of sound sensations and the basis of music follow from a study of the mechanism of the ear.

The first half of the second year should be devoted to *electricity*, including: conduction, in which the electric current and its continuity can well be introduced by the observations of electrolysis and in which conservation of energy is used to derive the laws of voltage, the derivative of energy with respect to electric quantity; electromagnetism, the kinetics of electricity, in which observations on induced voltage lead to the concept of magnetic flux, measured by its time-integral, and in which conservation of energy is used to derive the laws of magnetomotive force, the derivative of energy with respect to magnetic flux, and also the various formulas for mechanical force; electrostatics, the elasticity of electricity, in which dielectric current is shown to complete the circuit for conduction current, its time-integral measuring dielectric flux, and in which conservation of energy is again used to derive the formulas for mechanical forces; electric and magnetic fields, such as are derivable by superposition from those of point poles and straight-line poles. These fundamentals would then be applied to *electric oscillations and waves*, which are closely analogous to mechanical vibrations and waves and would be treated in corresponding ways: with free and forced oscillations should be introduced the concept of impedance and the use of vectors to represent it and the sinusoidal voltages and currents; wave pulses on wires serve to derive the formulas for velocity of propagation and natural impedance and for reflections at discontinuities; plane waves in space, their reflection and refraction, similarly treated, apply directly to radio, and anticipate the properties of light, including Fresnel's laws; radiation from a simple antenna or dipole requires only elementary calculus;[6] interference from an-

[6] A. Hazeltine, "Basic equations for electric and sound radiations," Trans. AIEE **57**, 774 (1938).

tenna arrays anticipates the diffraction grating. It will be appreciated that the simplification of the physical and mathematical methods has permitted the inclusion of some topics that are commonly reserved for upper-class and graduate courses. Moreover, the form of presentation is in accord with the viewpoint of electrical engineers and, in particular, calls for rationalized units, preferably of the engineering meter-kilogram-second system. So the subsequent electrical engineering course is aided.

The final half year should be devoted to light and atomic physics. With *light*, the observations of reflection and refraction suggest the hypothesis of a wave; and its propagation through a vacuum suggests an *electromagnetic wave*. The precise agreement of the measured velocity with theory renders this hypothesis a practical certainty, with further confirmation by the phenomena of polarization and of interference. The theory of propagation through nonisotropic materials agrees with the observations of elliptic polarization and double refraction. All of the physics course up to this point has dealt with observations on *gross matter*. Now comes the time to explain them by an *atomic hypothesis:* first come the isolation of the electron and the measurement of its charge, by the Millikan oil-drop experiment, and then the isolation of atoms and the measurement of their masses, by a mass spectograph. At once we have a verification by Faraday's law of electrolysis and the value of the Faraday constant. A corollary of the same hypothesis leads to the kinetic theory of gases, verified by Avogadro's law (including the Avogadro number) and the laws of ideal gases (including the adiabatic exponent, at least for monatomic gases). Statistical ideas explain the significance of the second law of thermodynamics. The host of mechanical, thermal, electric and chemical phenomena receive qualitative or quantitative explanation: mass, cohesion and elasticity, friction, viscosity, surface tension, thermal expansion, evaporation and thermionic emission, heat and electric conduction, dielectric phenomena, magnetism, valence, crystal structure, and many more. Finally, a brief introduction to the quantum hypothesis is afforded by photoelectric and kindred phenomena.

Mechanism and *shop practice* are the engineering courses of the first year and go naturally together: the shop is the experimental laboratory for the mechanism course and teaches how things are made. Both rest on the geometry and kinematics of the early work in physics. Mechanism includes descriptive geometry, engineering structures and mechanisms, and elementary machine design, each with work in the classroom and in the drafting room. Facility in mechanical drawing is an important by-product, like the manipulative skills acquired in the experimental laboratories.

Mechanics is the engineering course of the second year and would include strength of materials and mechanics of fluids, both of which are built on the physics course of the first year and perfect and extend knowledge and skills previously acquired in a rather rapid survey. The work in the engineering laboratory brings the student to the realization of practical departures from ideal conditions and the need for judgment as well as a knowledge of theory.

Chemistry. A departure is suggested in deferring chemistry to the second year. At present a descriptive course in general chemistry is commonly given in the first year and is often merely a glorified high-school course, suited to students who entered without any preparation in chemistry. Such courses are handicapped because they have to include parts of physics not yet reached in the physics course—mass and density, the properties of gases, heat and electricity as it enters in the attraction of atoms and in electrolysis. The present suggestion raises the level of the chemistry course and allows it at once to aim at the heart of chemistry, which is *chemical equilibrium*. While it cannot wait for the physics course to reach the electron, its first semester is concurrent with the physics course in electricity; so the two courses aid each other. Chemistry should start with the hypothesis of atoms and ions, each consisting of a nucleus and electrons in certain relatively stable arrangements, as best illustrated by the behavior of solutions. A study of equilibrium in solutions naturally correlates with a laboratory course in qualitative analysis, followed by quantitative analysis by titration and electrolysis. The first year of the course would be devoted to inorganic chemistry; and the second year might be shared by organic chemistry (including biological as well as industrial processes) and physical chemistry (including thermodynamic theory and nuclear reactions), with appropriate laboratory exercises of a quantitative nature. Throughout the course, emphasis should be placed on principles rather than on industrial practice: industrial processes will often be out of date when the student

graduates, but they are never out of date as illustrations of principles.

Electrical engineering. With the background provided by the physics and mathematics courses, the student is ready for the study of both direct-current and alternating-current circuits and apparatus. Starting with stationary apparatus—reactors, capacitors and transformers—the course may proceed to the common types of rotating machine—direct-current, synchronous, induction and (more briefly) single-phase commutator. In each case, construction, operation and control would be studied, rather than design. Measuring equipment and other auxiliary apparatus would be included. The elements of power distribution and an introduction to communication systems and methods, including some electronics, would complete the course.

Mechanical engineering. With the introduction in the physics course and with the engineering courses of the first two years, the student has already progressed well into mechanical engineering, which would now be continued both in mechanics and in thermodynamics. The mechanics work might be largely applied kinetics, treating of such topics as gyroscopes, governors and the balancing of engines. The thermodynamics would include steam engines and turbines (together with boilers), internal combustion engines, air compressors, refrigeration, air conditioning, and the flow of compressible fluids.

There has been sufficient experience to indicate that the three-year undergraduate curriculum just outlined is practicable. In scope, it should compare favorably with present four-year curriculums.

The graduate course in the fourth year, leading to a master's degree, would be of the university type for those students who look forward to teaching or to fundamental research, while it would be essentially a professional course for industrial physicists or for engineers. Its curriculum would vary with the particular needs of the student, who might sometimes find it desirable to transfer to another college or university known for its strength in his chosen field. To suggest a specific curriculum would therefore not be appropriate.

Physics holds the key position in the curriculum: it leads into all the engineering courses, has close ties with the other sciences, and underlies modern life, whose interpretation is the first aim of the humanistic courses. While still a teacher of engineering, and long before I had any idea of becoming a physics teacher, I asserted that physics was the most important, and the most difficult to teach, of any subject in an engineering curriculum. After putting our own house in order, let physics teachers hasten the proposed reform of the curriculum. Let us exert the influence that will make us leaders in American education!

What are Lectures Good for?

J. J. G. McCue
*Hamilton College, Clinton, New York**

IT is apparent from recent articles[1–5] in this journal that considerable discontent with present methods of teaching physics exists. Although physics possesses in the highest degree both practical importance and intellectual value, it has but little appeal to the general body of students; it is reasonable to suppose, therefore, that it is not being well taught. To impute the blame to the students is manifestly to put the cart before the horse. The students are there—it is up to us to teach them, and to teach them as they are, meanwhile working for improvement in the schools from which they come.

Since lectures have been the most prominent feature of most physics courses heretofore, the lectures naturally come under suspicion. Hence it seems worth while to consider what lectures can do, and to compare this with what we have been trying to make them do. The following observations will, it is hoped, be regarded as self-evident by anyone who has had a few years of teaching experience. The writer feels, however, that they are not for that reason superfluous;

* Now at the Radiation Laboratory, Massachusetts Institute of Technology, Cambridge 39, Mass.

[1] AAAS Committee Survey, Am. J. Phys. **8**, 49 (1940).
[2] R. B. Abbott and H. H. Remmers, Am. J. Phys. **8**, 244 (1940).
[3] R. R. Hancox, Am. J. Phys. **9**, 371 (1941).
[4] O. Blüh, Am. J. Phys. **10**, 39 (1942).
[5] W. Weniger, Am. J. Phys. **12**, 98 (1944).

they are intended to stimulate a stock-taking, or an examination of conscience, preparatory to the seemingly inevitable postwar recasting of the introductory courses. Much of what is said applies also to more advanced courses.

In the article by Weniger[5] it is pointed out that the lecture is a very inefficient way of transmitting factual knowledge. This defect is to some extent neutralized by the consideration that a great many facts can be presented in a one-hour lecture, so that even though a student retains only 10 percent of them, he may learn more facts *per hour* than he could, for example, in the laboratory. Factual knowledge, however, can readily be acquired through reading. We all know that a student normally learns factual lecture material by taking notes (of uncertain accuracy and completeness) and then reading them later. Surely such business can be transacted better by means of a textbook, or lithographed or mimeographed notes prepared by the professor.

The factual aspect of physics is neither the difficult nor the rewarding one, however; the concepts and the reasoning form the really nutritious, albeit not easily digestible, meat that is supported by the skeleton of fact. The facts can be transmitted in writing; the legitimate function of the lectures is to teach concepts and guide reasoning, and incidentally to arouse and *sustain* interest. In the demonstration lecture, the teacher of physics has a resource that cannot be matched in any other subject. Only the lecturer in chemistry can vie with him, but even he cannot rival the variety and the element of surprise that are so easily attainable in physics. Demonstration lectures are ideally adapted to sustaining interest in the subject and to illustrating the concepts and persuading the student of their validity. Seeing is believing. Administrators are doubtless aware of the merit of such lectures; they must be brought to recognize that the preparation of a good one is a very time-consuming business.

We have perhaps been less conscious of the other legitimate function of lectures, the guiding of physical reasoning. Regardless of demonstrations, the lecture provides the student with an opportunity to learn, as it were by *contagion*, how a master of the subject does his thinking about it. Obvious implications of this fact are that the lecturer should really be a master of the subject under discussion, that his thinking should be *sufficiently lucid to serve as a model* for the student, and that the lecturer and student should be in sufficiently close intellectual contact so that the contagion can occur. The writer believes that the last condition is really important and should not be dismissed as mysticism, even though the actual means of satisfying it are somewhat hard to come to grips with. Clearly the transmission of the teacher's attitude by contagion is not limited to matters of technic such as problem solving; the lecturer can expose students to a scale of values, to a differentiation between what is essential and what is secondary, more compellingly than the printed page ever can. These considerations apply equally to the teaching of freshmen and of graduate students.

In sum, the function of lecturing is not to teach the student facts, but to whet his appetite for them, to impart to him enthusiasm and perspective which will take him to the library or laboratory ready to spend his time profitably there, aided by a recognition of what is fundamental and what incidental. If this view is correct, retrospection of our prewar practices will in all too many cases give small cause for satisfaction.

The chief reason for our trying to make lectures do things they are not suited for is not far to seek: the procedures suggested here involve more work for the lecturer. Nevertheless it is reasonable to believe that if undergraduate courses in physics, and particularly the introductory one, are given by a man of proper training and temperament, whose schedule allows him sufficient time to do the job well, the department will be rewarded by improved enrolment and improved results. The fresh start after the war will afford to teachers and administrators a unique opportunity to reorganize an enterprise that has shown marked signs of unsoundness. Let us make the most of it!

A textbook must be exceptionally bad if it is not more intelligible than the majority of notes made by students. . . . The proper function of lectures is not to give a student all the information he needs, but to rouse his enthusiasm so that he will gather knowledge himself, perhaps under difficulties.—J. J. THOMSON.

A Convenient Virtual Image Locator for Elementary Optics

PAUL K. TAYLOR
Oberlin College, Oberlin, Ohio

THE time-honored method of locating virtual images in the college physics laboratory has been that involving the use of parallax. When no relative motion between the image and a "finder" pin is observed as the observer's head is moved in a direction transverse to that of the light, the position of the image is identical with that of the "finder" pin. This method is satisfactory in concept, but in actual practice often gives an elementary student very poor results and literally a headache because of eyestrain. To overcome both of these undesirable results and also to shorten the time involved in making relatively accurate determinations of the positions of virtual images of simple lenses, a virtual image locator was designed by the author and put to use in the optics section of the general physics course at Oberlin College.

The instrument consists merely of a supported metal tube about 4 cm in diameter and 30 cm long within which are mounted a double convex lens of 15-cm focal length and a circular ground-glass plate. These are equidistant from the opposite ends of the tube and are separated by a distance of 25 cm. Figure 1 shows the virtual image locator in the foreground. Slots in the front and rear faces of the instrument mounting pass the meter stick which, equipped with suitable end supports, constitutes the optical bench. The position of the image locator is taken as the reading of the meter stick where it enters that face of the mounting which is toward the object.

Other apparatus shown in Fig. 1 are a mounted diverging lens, the lamp box with the illuminated letter **F** acting as an optical object and, to the rear, a "focal-length box" for direct determination of the focal lengths of converging lenses. One of these lenses is shown suspended from the short metric ruler. This box has an aperture in the face just behind the suspended lens and a large ground glass image plate in the opposite face. Since the zero point of the metric ruler is mounted flush with the image plate, the focal length of the converging lens is read directly from the ruler after the lens position has been adjusted to give a clear image on the plate of a distant object or outdoor scene.

Acting as a fixed-focus "eye," the virtual image locator will have a sharply focused *real* image on its ground glass plate only when the source of light—be it a real object, a real image or a virtual image—is at *one particular distance from* the chosen reference point on the locator since the distance between its lens and image plate is invariable. This distance D from source to image locator, hereafter called its "focusing distance," is determined by moving the locator toward the illuminated **F** in the lamp box and observing the distance between the front face of the locator mounting and the **F** when a clear image of the latter appears on the locator's ground glass plate. Figure 2 will illustrate the determination of this focusing distance D. Though all the illustrations here included show D terminating at the lens of the image locator, it may equally well be measured to any fixed position on the mounting of the instrument, such as the front face, which is the position used in practice. The lower diagram of Fig. 2 shows the image locator at a distance D to the right of a real image which it has "located."

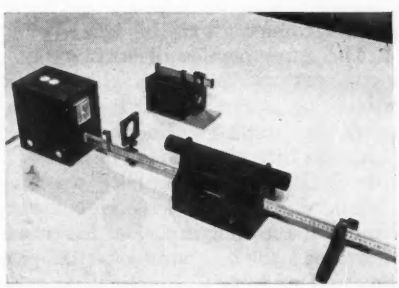


FIG. 1. Optical bench equipped with virtual image locator. In the rear is a "focal-length box," for direct determination of the focal lengths of converging lenses.

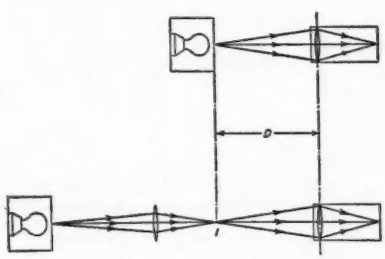


FIG. 2. Illustrating the method of determining the focusing distance D.

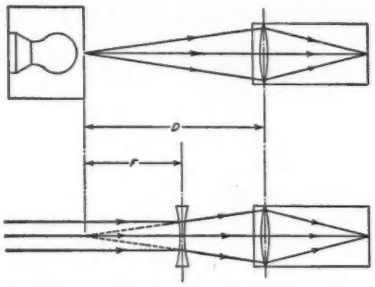


FIG. 3. Method of locating the principal focus of a diverging lens.

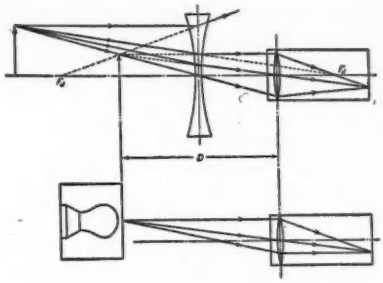


FIG. 5. Formation and location of images of object-points not on the common axis of lens and image locator.

This use is superfluous. It does, however, serve as an introduction of the image locator to the student who has just finished focusing the real image I on a white screen.

The lower part of Fig. 3 shows the image locator at a distance D to the right of the *virtual* image, which is at the principal focus of a *diverging* lens, since the incident light rays are parallel to one another and to the lens axis. Subtraction of the distance D from the scale reading of the position of the image locator therefore gives the position of the image. The principal focal length is then found, obviously, by taking the difference between the positions of virtual image and lens. This location of the principal focus of a concave lens is accomplished in the laboratory experiment as follows. The concave lens is mounted in the socket permanently fastened at the 10-cm mark of the meter stick as illustrated in Fig. 1. The stick, lens and image locator are next removed from the lamp box. The locator is supported from underneath by the fingers of the left hand. With the lens toward the window, the meter stick is pulled through the slots in the locator mounting until a clear image of a distant tree or building is seen on the glass plate. The locator is then in the position shown in the lower part of Fig. 3. The upper part of Fig. 3 again shows the determination of the focusing distance D for comparison.

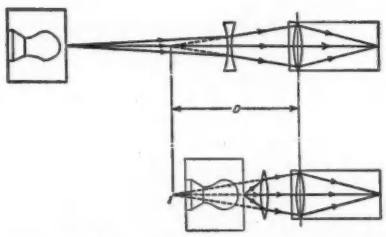


FIG. 4. *Above:* object at a finite distance from a diverging lens. *Below:* object nearer to a converging lens than its principal focus.

The only change in the upper part of Fig. 4 from the lower part of Fig. 3 is that the optical object has "moved in" from infinity to a point which in practice is from 20 to 50 cm from the diverging lens if the principal focal length of the latter is about -20 cm. The virtual image, acting as the source of light for the image locator, is now also closer to the diverging lens than is its principal focus.

In the lower part of Fig. 4, the illuminated object **F** is closer to a *converging* lens than its principal focus. In this case, the rays of light from object to lens are too divergent to be rendered converging or even parallel by the lens and so diverge from the virtual image I to the lens of the image locator. As before, the virtual images in both upper and lower diagrams of Fig. 4 are located by subtracting the focusing distance D from the observed scale reading at that position of the image locator which gives a clear image of **F** on its ground glass plate.

Figure 5 shows ray diagrams of the formation and location of images of points of the object not on the common axis of lens and image locator. The upper portion illustrates that, for a clear image to appear on its glass plate, the locator must be placed so that its face is at the distance D (cm) to the right of the virtual image formed by the diverging lens. The arrow represents the illuminated **F** actually used as the object. The size of the lens is considerably exaggerated in the diagram. The lower part of the figure shows the comparable situation using lamp box and image finder only. Figure 5 also illustrates that an inverted image on the glass plate of the locator indicates that the image being located is upright.

TABLE I. Use of the virtual image locator.

Positions (cm) Object F	Image finder	Focusing distance D (cm)
2.0	40.9, 41.2, 41.0, 41.1, 41.1, 40.9, 41.2, 41.2, avg. 41.1	39.1

Positions (cm) Image locator	Avg. dev.	Image	Lens	Object	Distances (cm) u	v	$uv/(u+v)$
Diverging lens (double concave). Time to secure data, 10 min							
29.1, 29.1, 29.0, 29.2, 29.0, *29.1*	0.07	−10.0	10.0	Distant Scene	∞	−20.0(f)	
47.6, 47.3, 47.4, 47.5, 47.6, *47.5*	.10	8.4	17.0	2.0	15.0	−8.6	−20.2
51.3, 50.9, 51.3, 51.0, 51.2, *51.1*	.16	12.0	22.0	2.0	20.0	−10.0	−20.0
58.9, 58.9, 59.0, 59.2, 59.3, *59.1*	.16	20.0	32.0	2.0	30.0	−12.0	−20.0
Converging lens (double convex). Time, 20 min (due to the turning of lens)							
105.9, 106.1, 105.7, 105.6, 105.5, *105.8*	0.20	66.7	34.6	2.0	32.6	32.1	16.2
21.1, 21.5, (turned) 24.5, 24.2, *22.8*	.20 .15	−16.3	12.5	2.0	10.5	−28.8	16.5
32.4, 32.8, (turned) 34.0, 34.5, *33.4*	.20 .25	−5.7	10.0	2.0	8.0	−15.7	16.3

Principal focal length (direct measurement)
15.9, 15.8, 16.0 (turned)
16.4, 16.6, 16.5; average, 16.2 cm

Typical results obtained by use of the virtual image locator are shown in Table I. The italicized numerals in the column of image locator positions denote average values. The average deviation from the means for all settings for the diverging lens was 0.12 cm. The computed values of the principal focal length f in the last column are consistent, and their average differs from the directly measured value of −20.0 cm by only 0.3 percent. It is to be noted that the total time taken to obtain all data for the diverging lens was only 10 min.

It was discovered that the converging lens used was not quite symmetrically mounted. This made it necessary to take measurements with each lens face in turn toward the object, which accounts for the longer time (20 min) required and for the somewhat larger deviations from mean settings in column 2 of the lower part of the table. Direct measurements of the principal focal length for the converging lens, shown in the bottom row of this table, were made with the "focal length box" previously mentioned.

For a comparison of time required and accuracy obtained in the location of images by the virtual image locator and by the method of parallax, respectively, Table II is shown, exhibiting results obtained by the latter method with the same diverging lens as was used for the results in Table I. Deviations in position from mean settings are about three times as large in the parallax method as with virtual image locator (0.4 cm *versus* 0.12 cm). Moreover a time of 49 min for 33 readings, or 1.5 min per reading, was required for careful settings by the parallax method. This compares with 10 minutes for 20 settings, or 0.5 min per setting, with the virtual image locator.

TABLE II. Location of virtual images by parallax (for comparison). Same diverging lens as in Table I. Positions and distances in centimeters.

Chosen positions object	image	Determined position of lens	Avg. dev.	Number of settings	Time (min)	u	v	$uv/(u+v)$
2	20	33.2	0.4	4	5	31.2	−13.2	−22.7
2	10	19.8	.4	4	5	17.8	−9.8	−21.8
50	60	69.4	.4	8	9	19.4	−9.4	−18.2
55	60	*66.4	.7	8	10	11.4	−6.4	−14.6
∞	60	79.8	.2	9	20	∞	f = −19.8	avg. = −19.3

* Acute eyestrain developed here.

Thus Tables I and II show that, for the measurements therein exhibited, the use of the virtual image locator gave, in one-third of the time, results which were three times as accurate as those obtained by the method of parallax. It should be added that eyestrain experienced by the author in obtaining the data shown was severe when using the parallax method but was hardly noticeable during the use of the image locator. In view of the shorter time and effort involved in the location of virtual images, the much greater accuracy obtainable, and the often expressed student interest and pleasure in its use, it is felt that this virtual image locator is a worthy addition to the elementary physics laboratory equipment.

The author wishes to express his appreciation to other members of the Department of Physics at Oberlin College for assistance given in designing the virtual image locator; to Dr. G. K. Schoepfle who designed the illuminated object housing and the focal-length box; to Mr. Harlan E. Hurd, of the departmental instrument shop, and his assistants who constructed sets of these instruments for test and laboratory use; to Mr. Sam Oi who made the drawings; and to Mr. Arthur Princehorn, the college photographer, who photographed the apparatus and drawings.

Ball Pendulum Impact Experiments

JOHN SATTERLY
University of Toronto, Toronto, Canada

INTERESTING experiments on the impact of elastic spheres have been recently described in this journal.[1] I should like to describe what we have done in Toronto on this subject; it may draw forth the experiences of others. For years our elementary classes have worked with two-ball experiments where the balls hang as (simple) pendulums from the ceiling, with wires parallel and centers of balls on the same level, so that the balls when at rest are just in contact. We have dealt only with head-on collisions, neglecting all that go askew, and find that if care is taken there are not sufficient skew collisions to justify the trouble of using bifilar suspensions.

Figure 1 represents one of our arrangements. Here M_2, M_1 represent cast-iron balls of masses about 32 and 9 lb, respectively. The fine steel suspension wires are fastened to the ceiling, and the length from the supports to the centers of the balls is 350 cm, or 11.5 ft. The pendulum periods are 3.75 sec, giving plenty of time for making observations. With such lengths it is possible to draw the balls aside as much as 50 cm and yet be able to assume, within much less than 1 percent, that the velocities acquired when the balls reach their lowest positions under the action of gravity alone are proportional to their initial displacements. The displacements before and after collision are measured with wooden brackets K, K, which, manipulated by hand, slide alongside a 2-m scale SS. Either (i) one ball is allowed to remain at rest and the other ball pulled aside and released, making impact at the lowest point, and the subsequent displacements are measured, or (ii) both balls are drawn aside and released simultaneously so that here also the collision occurs at the lowest positions.[2] It takes nimble action with the brackets to measure correctly the displacements after the first impact, but it can be done. However, in most experiments several observations are taken and the mean displacements computed. With a heavy ball a bracket may be put just within the estimated displacement and the ball allowed to push it out.

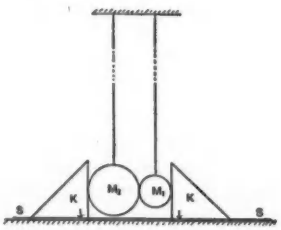


FIG. 1. Arrangement for impact experiments with two balls.

In our classes we have two two-ball experiments on impact. In the first the balls are equal in mass, and the experiment consists in proving the theorem of the conservation of momentum. In the second experiment the balls are of unequal mass (as in Fig. 1), and here the theorem of conservation of momentum is assumed and the experiment used to find the ratio of the masses. From each experiment the coefficient of restitution is calculated. It is constant for the same pair of balls if impact occurs at the same place, and I have recently put paint-marks on the balls to show which is the normal position of the ball (when at rest) so that rotation of the balls and thereby impacts occurring at different spots may be avoided.

The equations we use are

$$m_2u_2+m_1u_1=m_2v_2+m_1v_1, \qquad (1)$$

$$e(u_2-u_1)=-v_2+v_1. \qquad (2)$$

Here m_2, m_1 are the masses of the balls; u_2, u_1 are the respective velocities before impact; v_2, v_1 are the respective velocities after impact; and e is the coefficient of restitution for the two balls. From Eq. (2), which is Newton's empirical law of restitution, we see that e is equal to the relative velocity of separation divided by the relative velocity of approach.

It takes time for some students to master the sign-conventions for velocities, and this experiment affords practice. In the case of perfectly elastic balls of equal mass, if one ball is initially at rest and the second falls and hits it, the second ball remains at rest after impact. But in the case of iron balls of equal mass, the second ball does

[1] H. B. Lemon, **3**, 36 (1935); W. W. Sleator, **5**, 6 (1937); S. Chapman, **9**, 6 (1941).

[2] Even up to an amplitude of 50/350 rad, the period of a pendulum does not exceed 1.002 times the period of the same pendulum with an infinitely small amplitude.

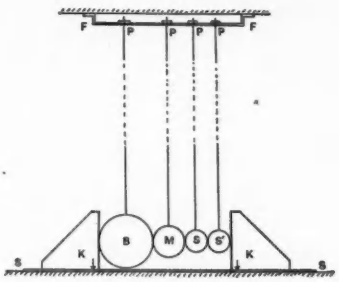


FIG. 2. Arrangement for impact experiments with four balls.

not remain at rest; it follows on, and the students often err here unless they are nimble with mind and bracket. Another mistake students often make is to wait for a second collision before setting the brackets for the measurement of the displacements.

Chapman's paper[1] prompted me to set up a three-ball and later a four-ball impact experiment to test what happens in such cases, where impacts seem simultaneous. My final arrangement is shown in Fig. 2. All the balls are of cast iron and were obtained from the workshop. Balls B and M have masses of 31.8 and 8.93 lb, respectively; they had been in use in impact experiments for many years and therefore may be quite hardened around their equatorial regions. The two small balls S and S' are equal in mass, 3.97 lb; they were rough-cast and had not previously been used in impact experiments. The masses were measured with spring balances. The mass ratios are:

$$B/M=3.56, \quad M/S=M/S'=2.25,$$
$$B/S=B/S'=8.01.$$

The balls hang by steel wires of diameter 0.7 mm from points of support P, P, $\cdots$, which are adjustable along a slot in a metal frame FF that is screwed to the ceiling. The line of centers of the balls is adjusted to be horizontal, and parallel to the edge of the bench. The supports P may be adjusted along the slot and then clamped so that the balls may hang either just in contact or with measurable gaps between them.

If a ball of mass m_2 traveling with speed u_2 hits head-on a ball of mass m_1 initially at rest and if e_2 is the coefficient of restitution between the two balls, the speed v_1 of m_1 after the collision is obtained by solving the equations

$$m_2u_2=m_2v_2+m_1v_1, \tag{3}$$
$$e_2u_2=-v_2+v_1; \tag{4}$$

thus

$$v_1=[m_2/(m_2+m_1)](1+e_2)u_2.$$

If m_1 subsequently hits head-on a ball of mass m originally at rest and e_1 is the coefficient of restitution between these two balls, the speed v of m after the collision is given by

$$v=[m_1/(m_1+m)](1+e_1)v_1$$
$$=\left(\frac{m_2}{m_2+m_1}\right)\left(\frac{m_1}{m_1+m}\right)(1+e_2)(1+e_1)u_2.$$

If m in turn hits head-on a ball of mass m_0, originally at rest, and e_0 is the coefficient of restitution, the speed v_0 of m_0 after the collision is given by

$$v_0=\left(\frac{m_2}{m_2+m_1}\right)\left(\frac{m_1}{m_1+m}\right)\left(\frac{m}{m+m_0}\right)$$
$$\times(1+e_2)(1+e_1)(1+e_0)u_2, \tag{5}$$

and so on if further collisions are made.

Two-Ball Experiments

Impact experiments were first made with the balls in pairs, B and M, M and S, S and S', to check the mass ratios and the values of the respective coefficients of restitution. The two balls not in use at any time were, of course, drawn out of the way. The values obtained for the coefficients were: for B and M, 0.89; for M and S, 0.83 or 0.82; for S and S', 0.77+. These values are in approximate accordance with the aforementioned histories of the balls.[3] Many experiments showed that the values of e did not change measurably with different velocities before and after impact.

Affixing an excise stamp from a packet of tobacco to a ball at the point of impact lowered the coefficient for B and M from 0.89 to 0.71, and for M and S from 0.83 to 0.66.

Illustrative example involving M and S.—Ball M was drawn back 50.0 cm and, with S at rest, was released. After impact M went on 22.0 cm and S 63.3 cm. Expressed in arbitrary units, the momentum before collision was 8.93×50.0, or 447 units, and that after collision was $(8.93\times22.0)+(3.97\times63.3)=448$ units, a confirmation of Eq. (3). By Eq. (4) the coefficient of restitution between M and S was $(63.3-22.0)/50.0=0.83$.

Three-Ball Experiments

Suppose that three balls are just in contact when at rest and that one of the outside balls is drawn aside and released. The problem is to calculate the velocities after impact. Experiment shows that upon impact the balls which were originally at rest separate from one another; therefore, they cannot be treated as a single body during the impact. Also at first glance one can

[3] These experiments have been continued and, as a result of the repeated impacts, the value of e for S and S' has now risen to 0.83.

hardly apply Newton's law of restitution to the two balls originally at rest.

But the experimentally determined velocities do agree very nearly with the velocities calculated on the supposition that the two balls at rest and in contact can be treated as if not in contact. For example, suppose that B falls and hits M, which is in contact with S. Calculate the velocity of M on the assumption that S is absent. Then suppose that M with this velocity hits S. The computed velocity of S agrees with the experimental value. In the calculations it is convenient to take the velocity of the "striker" ball as positive.

This result was checked also by shifting the supports at the ceiling, so that there was a small gap of about 3 mm between M and S, whereupon the same experimental results were obtained. In this case, B, M and S eventually move in the same direction. The values of the calculated velocities must, however, be watched; for if M's velocity after hitting S is less than B's velocity after hitting M, B will catch up with M and hit it again and M may again hit S.

If S were drawn back and allowed to fall on M (and B), both S and M would rebound; and if we work out the successive impacts of S on M, M on B and M back again on S, the calculated velocities will be found to be in agreement with the experimental ones. Hence we see that the light contact existing between the hanging balls does not stop the balls from behaving as if out of contact. Possibly the roughness of the surfaces has something to do with this although Chapman (p. 358) indicates that a step-by-step calculation gives true results in his experiments with smooth (elastic) steel balls.

Example 1.—Ball B is drawn back 25.2 cm and, when released, falls and hits M, which is in contact with S; B, M and S are observed to go forward 13.9, 18.8 and 46.6 cm, respectively. The momentum before collision is 31.8×25.2, or 801 units. The momentum after collision is $(31.8\times13.9)+(8.93\times18.8)+(3.97\times46.6)$, or 795 units. Thus momentum is conserved. Next calculate the velocities. Let v_2, v_1 be the speeds of B and M after first collision (S supposed absent). The equations are

$$31.8\times25.2=31.8v_2+8.93v_1,$$
$$0.89\times25.2=-v_2+v_1.$$

Solving these, $v_2=14.8$ units and $v_1=37.2$ units. Now M hits S. Let again v_2 and v_1 denote the velocities after collision:

$$8.93\times37.2=8.93v_2+3.97v_1,$$
$$0.825\times37.2=-v_2+v_1.$$

Solving these, $v_2=16.3$ units and $v_1=47.0$ units, and B does not strike M again. As Table I shows, the agreement is fairly good. Or we may calculate the speed of S by Eq. (5); thus,

$$\frac{31.8}{31.8+8.93}\times\frac{8.93}{8.93+3.97}\times1.89\times1.825\times25.2=46.9 \text{ units.}$$

TABLE I. Results for Example 1.

Velocity	Before Impact B	After Impact B	M	S
Actual	25.2	13.9	18.8	46.6
Calculated		14.8	16.3	47.0

Example 2.—Ball S is drawn back 40.0 cm and released. After collision S rebounds 11.6 cm, M rebounds 10.2 cm and B goes on 9.2 cm. The momentum before collision is 3.97×40, or 159 units; that after collision is $-(3.97\times11.6)-(8.93\times10.2)+(31.8\times9.2)$, or 155 units.

(*a*) Collision between S and M:

$$3.97\times40.0=3.97v_2+8.93v_1,$$
$$0.82\times40.0=-v_2+v_1;$$

whence $v_1=22.4$ units for M, and $v_2=-10.4$ units for S.

(*b*) Collision between M and B:

$$8.93\times22.4=8.93v_2+31.8v_1,$$
$$0.89\times22.4=-v_2+v_1;$$

whence $v_1=9.3$ units for B, and $v_2=10.6$ units for M.

(*c*) Ball M rebounding from B catches up with S and hits it again:

$$(8.93\times10.6)+(3.97\times10.4)=8.93v_2+3.97v_1,$$
$$0.82(10.6-10.4)=-v_2+v_1;$$

whence $v_1=11.7$ for S, and $v_2=10.1$ for M. Table II shows that the agreement is good.

Many such experiments were performed. That the velocities can be calculated by a step-by-step procedure is fairly substantiated by the experimental evidence.

Four-Ball Experiments

The four balls hang as in Fig. 2 with the wires parallel the balls in contact and their centers on the same level.

Example 3.—Ball B is withdrawn 30.0 cm and released. After the collision B is observed to go on 15.3 cm; M, 16.5 cm; S, 25.0 cm; and S', 52.5 cm. The momentum before collision is 31.8×30.0, or 954 units; the momentum after collision is $(31.8\times15.3)+(8.93\times16.5)+(3.97\times25.0)$

TABLE II. Results for Example 2.

Velocity	Before Impact S	After Impact S	M	B
Actual	40	−11.6	−10.2	9.2
Calculated		−11.7	−10.1	9.3

$+(3.97\times52.5)$, or 942 units, in good agreement with the conservation law. Calculations of the successive steps:

(*a*) Ball B strikes M:

$$31.8\times30.0=31.8v_2+8.93v_1,$$
$$0.89\times30.0=-v_2+v_1;$$

whence $v_2=17.5$ units for B, and $v_1=44.3$ units for M.

(*b*) Ball M strikes S:

$$8.93\times44.3=8.93v_2+3.97v_1,$$
$$0.83\times44.3=-v_2+v_1;$$

whence $v_2=19.3$ units for M, and $v_1=56.1$ units for S.

(*c*) Ball S strikes S':

$$3.97\times56.1=3.97v_2+3.97v_1,$$
$$0.775\times56.1=-v_2+v_1;$$

whence $v_2=6.3$ units for S, and $v_1=49.8$ units for S'.

(*d*) Ball M is traveling faster than S, so M hits S again:

$$(8.93\times19.3)+(3.97\times6.3)=8.93v_2+3.97v_1,$$
$$0.83(19.3-6.3)=-v_2+v_1;$$

whence $v_2=11.9$ units for M, and $v_1=22.7$ units for S.

(*e*) Ball B is now traveling faster than M, so B hits M again:

$$(31.8\times17.5)+(8.93\times11.9)=31.8v_2+8.93v_1,$$
$$0.89(17.5-11.9)=-v_2+v_1;$$

whence $v_2=15.2$ units for B, and $v=20.0$ units for M.

From Table III the agreement is seen to be fairly good. The approximate values of the e's may be responsible for the discrepancy found for M. With only one experimenter, the task of measurements is not easy. A quick ear helps to detect some of these successive or multiple collisions better than the eye.

Application of Eq. (5) for successive contacts is allowable for S' in this case; it gives for the speed of S',

$$\frac{31.8}{31.8+8.93}\times\frac{8.93}{8.93+3.97}\times\frac{3.97}{3.97+3.97}\times1.89\times1.83\times1.775\times30.0=49.5 \text{ units},$$

as compared with 49.8 units in Table III.

Effect of small gaps.—The supports P, P, P were moved to give gaps of about 3 mm between M and S and between S and S'. Ball B was then drawn back the same distance as before and the displacements after collisions were measured. They were 15.25 cm for B, 19.2 cm for M 23.0 cm for S and 50.4 cm for S', in good agreement with calculated displacements, perhaps emphasizing the discrepancy of the contact experimental results.

TABLE III. Results for Example 3.

Velocity	Before Impact	After Impact			
	B	B	M	S	S'
Actual	30.0	15.3	16.5	25.0	52.5
Calculated		15.2	20.0	22.7	49.8

Example 4.—When the smallest ball S' is drawn back 40.0 cm and allowed to fall, S', S and M rebound distances of 13.3, 10.0 and 3.4 cm, respectively, and B goes on 8.65 cm. The momentum before collision is $3.97\times40.0=159$ units; that after collision is $-(3.97\times13.3)-(3.97\times10.0)-(8.93\times3.4)+(31.8\times8.65)$, or 152 units. The calculations are lettered in sequences of time as follows:

(*a*) S' hits S

(*b*) S hits M

(*c*1) S rebounds and hits S' (*c*2) M hits B

(*d*) M rebounds and hits S

(*e*) S hits S' again

Taking the direction of u_2 as positive in each case, we have:

(*a*) Ball S' hits S:

$$3.97\times40=3.97v_2+3.97v_1,$$
$$0.775\times40=-v_2+v_1;$$

whence for S', $v_2=4.5$ units; and for S, $v_1=35.5$ units.

(*b*) Ball S hits M:

$$3.97\times35.5=3.97v_1+8.93v_1,$$
$$0.83\times35.5=-v_2+v_1;$$

whence for S, $v_2=-9.5$ units (a rebound); and for M, $v_1=20.0$ units (forward).

(*c*1) Ball S rebounding hits S' which is traveling forward:

$$(3.97\times9.5)-(3.97\times4.5)=3.97v_2+3.97v_1,$$
$$0.775(9.5+4.5)=-v_2+v_1;$$

whence for S, $v_2=-2.9$ units (a rebound); and for S', $v_1=7.9$ units.

(*c*2) Ball M hits B:

$$8.93\times20.0=8.93v_2+31.8v_1,$$
$$0.89\times20.0=-v_2+v_1;$$

whence for M, $v_2=9.4$ units (a rebound); and for B, $v_1=8.4$ units.

(*d*) Ball M on the rebound from B hits S on the rebound from S':

$$(8.93\times9.4)-(3.97\times2.9)=8.93v_2+3.97v_1,$$
$$0.83(9.4+3.9)=-v_2+v_1;$$

whence for M, $v_2=2.5$ units; and for S, $v_1=12.7$ units.

(*e*) Ball S catches up with S' and hits it again:

$$(3.97\times12.7)+(3.97\times7.9)=3.97v_2+3.97v_1,$$
$$0.775(12.9-7.9)=-v_2+v_1;$$

whence for S, $v_2=8.4$ units; and for S', $v_1=12.3$ units. There are no more impacts. Affixing the signs, we can now compare experiment and calculation (Table IV). The agreement is fair. As a check on the calculated results we can compute the momentum after collision; it is $-(3.97\times12.3)-(3.97\times8.4)-(8.93\times2.5)+(31.8\times8.4)=163$ units, as compared with 159 units before collision.

Example 5.—This last experiment was repeated, but with S' now drawn back 50.0 cm. The results only are given (Table V). Again the agreement is fair.

TABLE IV. Results for Example 4.

Velocity	Before Impact S'	After impact S'	S	M	B
Actual	40.0	13.3	10.0	3.4	8.6
Calculated		12.3	8.4	2.5	8.4

TABLE V. Results for Example 5.

Velocity	Before Impact S'	After Impact S'	S	M	B
Actual	50.0	−16.6	−12.0	−4.6	10.9
Calculated		−15.8	−10.3	−3.5	10.5

Summary and Conclusions

When balls hang lightly in contact and by the withdrawal of an end ball and subsequent release impacts occur through the succession of balls, the collisions may be worked out and the velocities calculated on the supposition that the balls are not in contact but are free to move for a very short time. In the calculations one must watch for more than one collision between a pair of balls. This is in opposition to the statement often made that the end ball will be given a larger velocity if there are gaps between the balls than if the balls hang in contact.

We thus see that in the familiar lecture experiment where a number n of perfectly elastic balls of equal mass hang lightly in contact from bifilar suspensions, if an end ball is withdrawn and released, the ball at the other end flies off with an equal velocity and the $n-1$ balls remain at rest. If two balls are drawn back equally and released simultaneously, the preceding statement applies to each ball in quick succession so that two balls fly off from the far end with velocities equal to that of the falling balls and $n-2$ balls remain at rest. The same is true if m balls ($m<n$) be drawn back and released. Whereas, if an end ball has a mass double that of each of the other balls and it be drawn back and released, the sequence of collisions gives quite a different set of velocities.[4]

When two balls only are in use and one is at rest and the other withdrawn and released, the first few impacts take place at the lowest position of the balls. But gradually the position of the balls at impact passes away from the lowest position, indicating that the ball which goes out the farthest must start to fall back later than the other one does. Eventually the two balls swing together. Possibly this is because impact is not an instantaneous action. The same is true for more than two balls. No matter how the impacts be started, eventually all the balls swing together.

[4] See Chapman, ref. 1, third paragraph.

Meeting of the Illinois Chapter

THE Illinois Chapter of the American Association of Physics Teachers held a joint meeting with the Physics Section of the Illinois State Academy of Science at the University of Illinois, Urbana, Illinois, on May 5, 1945. About 30 persons were present. Luncheon was served at the University Men's Club.

The following program was presented.

Report of the annual meeting of the Association. R. F. PATON, *University of Illinois.*

Technical writing. DUANE ROLLER, *Wabash College.*

Physics and mathematics in the premedical program. L. I. BOCHSTAHLER, *Northwestern University.*

Advantages and disadvantages of student projects in shop physics. P. E. MARTIN, *Wheaton College.*

The time loss at starting for a sprinter. F. GILES, student, *Wheaton College.*

Numbering and affixing name plates to physics equipment. S. NIXON, student, *Wheaton College.*

Lifetime of excited nuclei. M. GOLDHABER, *University of Illinois.*

Theoretical physics and pioneering research in Illinois minerals. R. J. PIERSOL, *Illinois State Geological Survey.*

R. F. PATON, *Secretary*

"Action Equals Reaction"—Even in Gyroscopes

PAUL ROOD
Western Michigan College of Education, Kalamazoo, Michigan

GYROSCOPES, in a multitude of different forms, have proved themselves indispensable in certain phases of the modern mechanized world. The ships of most of the navies and merchant fleets of the world get true bearings from the gyro compass; the gyro horizon and direction indicator are standard equipment in large airplanes; torpedoes, robot bombs, fire-control mechanisms—in fact, any instrument that must have in it an element holding a fixed direction in space, despite a variety of local motions of the frame on which it is mounted—make use of the gyroscope.[1]

Any device coming into as general use as this, which at the same time depends directly for its operation on a fundamental principle in mechanics, deserves an important place in the course in general college physics. Yet, out of seven recent standard textbooks, only one gives to the subject fairly adequate treatment—a chapter. Of the others, two give three paragraphs each, three give only one paragraph, and the last omits the subject entirely.

In most textbooks the explanations of gyroscopic behavior make use of various artifices which, while perfectly logical on the basis of the assumptions that are made (but not always stated), still leave the student with the feeling that the gyroscope is a mysterious thing not subject to the ordinary laws of motion. The following explanation of gyroscopic action is based on the concepts of force, torque and Newton's third law of motion.

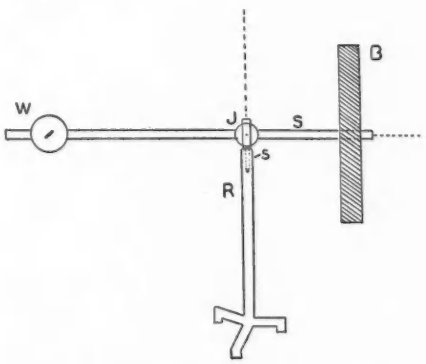


FIG. 1. A demonstration gyroscope.

A convenient apparatus commonly used for demonstrating how gyroscopes operate is shown in Fig. 1. A bicycle wheel *B*, the rim of which is weighted with lead, is mounted on one end of a shaft *S*, near the other end of which is clamped a movable counterweight *W*. The shaft is supported in a universal joint assembly *J*, consisting of a small iron sphere, through which the shaft passes, and an iron band fitted with two projecting studs which hold the sphere so that it and the shaft through it can turn around a horizontal axis. A vertical rod *s* supports the iron band and rests on a steel ball at the bottom of a cavity in the supporting rod *R*. The wheel mounted in this manner has freedom of motion around three mutually perpendicular axes and, when spun rapidly, exhibits the properties characteristic of a gyroscope.

From a qualitative point of view it makes no difference in the operation of the gyroscope if for the spinning wheel we imagine substituted a square frame made of hollow tubing filled with lead shot moving rapidly through the frame in the same direction the actual wheel is turning (see Fig. 2). The forces involved in the operation of this frame gyroscope differ only in magnitude from those involved in one that spins, and, therefore, an explanation of its action is immediately applicable to any gyroscope.

Owing to the combined effects of *W* and *B*, Fig. 2, a torque is applied to the gyro about an axis through the center of the universal joint assembly *J*, and perpendicular to both the axis of the frame and the vertical axis through *J*; let this torque be counterclockwise. While the entire gyro turns about the axis through *J*, the frame *B* rotates about a parallel axis through its center. Side *1* during the rotation takes successive positions *A*, *B* and *C* as shown in Fig. 3. A given piece of shot, for instance *x* in Fig. 3*A*, has just entered this side of the frame and is traveling with velocity *v* in the downward direction. An instant later the frame has turned to the position *B* and *x* is now traveling with the same speed but in a

[1] For an account of some naval applications of the gyroscope, see J. D. Riggin, Am. J. Phys. **12**, 291 (1944).

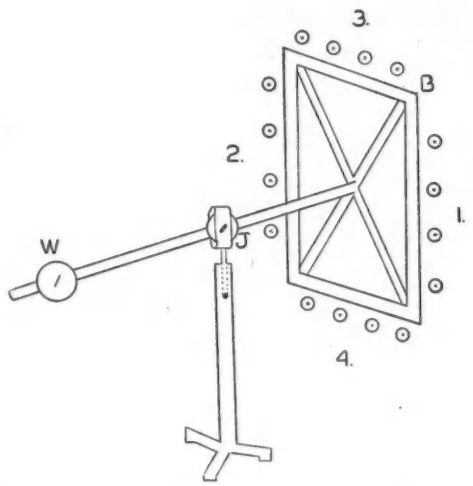


FIG. 2. A gyroscope with nonrotating hollow frame containing circulating shot.

new direction, downward and to the right. In position *C* the direction in which *x* is traveling has been shifted still farther to the right. Side *1* of the frame is continually exerting a force on particle *x* to accelerate it at right angles to the frame, just as the string holding a rotating weight continually exerts a force on the weight to accelerate it at right angles to the direction in which it is moving.

From Newton's third law of motion it is apparent that the particle during its passage down the tube must have been pushing on the tube with a force equal in magnitude and opposite in direction to that with which the tube pushes on it.[2] What is true of the given particle is equally true of all the shot which circulate through the tube, and the reaction force is multiplied by a factor equal to the number of shot in the tube. The direction of the reaction with respect to the frame is shown by the arrows in Fig. 3*A*, *B*, *C*. The consequent motion of side *1* is toward the reader in Fig. 2, as is indicated by the dots in the small circles.

In side *2* (Fig. 2) the particles travel upward. They are continually being urged to the left by the frame, as shown in Fig. 4, and in turn push the frame to the right as indicated by the arrows. The ensuing motion of this side is away from the reader and is indicated in Fig. 2 by the crosses in the small circles.

Sides *3* and *4* of the frame are not changing the directions of the particles in them when the turning around the horizontal axis alone is considered, and hence no forces of reaction are generated.

The combination of the reactions in sides *1* and *2* results in a torque that rotates the frame *B* about the vertical axis which lies in the plane of the frame through its center.

An analysis identical to that used for sides *1* and *2* will show that, owing to this turning about the vertical axis, side *3* is urged to the right (motion indicated by crosses) and side *4* to the left (motion indicated by dots). Considering this turning alone, we see that there are no forces in the vertical sections, since in them the directions of the particles are not being changed. The combination of the forces in sides *3* and *4* results in a torque that tends to turn the frame clockwise about a horizontal axis.

It is evident that the turning of the spin axis of a gyroscope about any axis sets up a torque

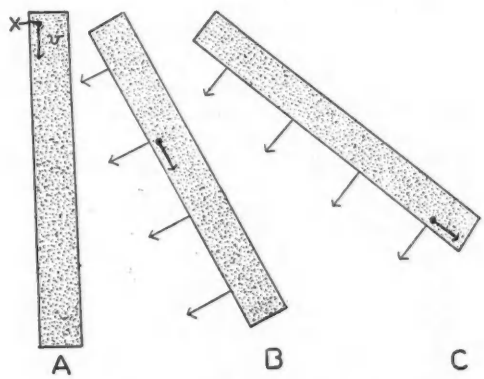


FIG. 3. Reaction forces acting on side *1* of frame gyro as it turns.

about an axis perpendicular to both the axis of spin and the axis of turning and that, reciprocally, the turning about this latter axis produces a torque about the first axis. No torque is necessary to maintain the turning itself. For instance, in Fig. 2 the "action" of the frame on all the particles in side *1* when it turns is opposite to the "action" of the frame on all the particles in side *2*.

[2] The force which the particle exerts on the tube is a "kinetic reaction." It is real only when considered as a force acting on the tube. It is not a force which acts on the particle itself and which, together with the opposite force applied by the tube, keeps the particle in equilibrium. The particle is not in equilibrium for it is being accelerated. See, for example, G. A. Lindsay, "Newton's third law," Am. J. Phys. **11**, 322 (1943).

The vector sum of these actions is zero and, therefore, no torque is necessary to maintain the turning. The turning may be considered as signaling the fact that a torque is acting about the perpendicular axis. Turning and torque about mutually perpendicular axes are the "Siamese twins" of gyroscopes. If the turning about either axis is prevented, the associated torque about the perpendicular axis disappears. To keep one without the other would be to deny the validity of the third law of motion.

It will be noted in the foregoing description and by referring to Fig. 2 that the torque set up about the horizontal axis, perpendicular to the axis of spin and the vertical, is opposite to the applied torque of W. It is also equal in magnitude to the applied torque; for, if it were not, the associated rate of turning about the vertical axis would immediately adjust itself to a value such that the resisting torque would be equal to the applied one. The name usually given to the resisting torque is "gyroscopic resistance." It is on account of this torque that gyroscopes may offer a stonewall resistance to attempts to turn them.

In the gyroscope of Fig. 1, if the wheel is started spinning and the weight is supported by the hand and then suddenly dropped, the axle will dip about an axis through J perpendicular to the plane of the paper. The consequent torque swings the axis clockwise in a horizontal plane with an increasing angular velocity since it meets no resisting torque. But this sets up a gyroscopic resistance which soon becomes larger than the applied torque, causing the axle to rise. When this happens the torque about the vertical is reversed in direction, causing the axle to move more slowly horizontally. As a result of the interplay of these torques, the end of the axis moves through a series of cycloidal loops—nutation—as is shown in Fig. 5. When the end is at any point E in its

FIG. 5. The cycloidal loops generated by the end of the axis of a gyro during nutation.

path it has a velocity p carrying it leftward and the velocity a, because of the combined effects of the applied torque and the gyroscopic resistance, carrying it downward. It takes a path that is tangent to the resultant of these two vectors. At F the velocity a is zero and the path is horizontal; at G the direction of a is reversed and p is the same as it was at E. At H the cycle starts over again.

If the bearings of the gyroscope were perfectly frictionless this nutation would continue indefinitely. In the actual case, friction soon damps it out. The resulting motion is a uniform angular velocity about the vertical axis. It is called "steady precession," and is a measure of the associated gyroscopic resistance which keeps the gyro from tipping. "Hurrying the precession" or "forcing the precession" may be accomplished by a touch of the finger since no torque is needed to cause steady precession, but this controls the gyroscopic resisting torque working against that causing tipping, and this resistance is very large if the wheel is heavy and is spinning rapidly.

To an extent determined by the bearing friction around the vertical axis, there will be a steady angular velocity of the gyro about a moving horizontal axis that is always perpendicular to the axis of spin. This will be small, since it depends on the torque caused by friction, which is small, and the axis will accordingly sag very slowly. This constant angular velocity is just as truly a steady precession as is that around the vertical axis and its "Siamese twin," the torque overcoming friction, is the associated gyroscopic resistance. This precession cannot easily be hurried since there is a large gyroscopic resistance opposing such an attempt.

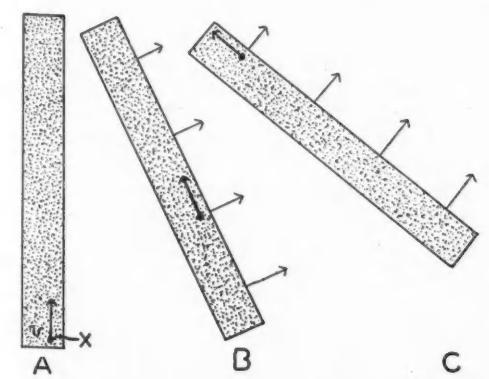


FIG. 4. Reaction forces acting on side *2* of frame gyro as it turns.

Homer Levi Dodge

Recipient of the 1944 Oersted Medal for Notable Contributions to the Teaching of Physics

The American Association of Physics Teachers has made to Doctor Homer Levi Dodge, President of Norwich University, the ninth of its annual awards for notable contributions to the teaching of physics. The addresses of recommendation and of presentation were made by Professor A. A. Knowlton, chairman of the Committee on Awards, and Professor Lloyd W. Taylor, President of the Association, in a ceremony held in McMillin Theatre, Columbia University, on January 20, 1945, during the fourteenth annual meeting.

ADDRESS OF RECOMMENDATION BY PROFESSOR A. A. KNOWLTON

THERE are many ways in which one may contribute to the betterment of physics teaching. In selecting the Oersted Medalist of the current year the committee recognized that the man chosen has made contributions in at least three of these ways.

He has been an original and stimulating classroom instructor; as the head of a department he was able so to select and organize his staff as to multiply his own classroom effectiveness many fold; because of a breadth of interest which acknowledges neither departmental nor institutional limits, his influence has spread so widely as to make him in effect a public relations counsel for all physicists.

DR. HOMER L. DODGE was graduated from Colgate University in 1910 and went at once to the University of Iowa where he remained for nine years as graduate student, instructor and associate professor. We have testimony from no less an authority than the president of the association (Dr. Taylor) as to the high quality of his instruction during this period. Both in his undergraduate days and while working as student and member of the department at Iowa he was under the influence of men—C. D. CHILD at Colgate and G. W. STEWART at Iowa—who were stimulating teachers and productive investigators. His steadfast belief that research and teaching are compatible activities may well be a product of these associations.

After an interlude spent in war work he went to the University of Oklahoma as head of the department of physics. One would like to recite in detail the story of the growth of this department in both numbers and effectiveness under his direction; but this would take too long. All that can, or perhaps need be, said is that he put into practice the ideals previously developed in which research and teaching were carried on by the

same staff. He selected his staff with this in mind and then expected all members of the department to contribute to the aliveness and effectiveness of the teaching program, each after his own fashion and by his own methods.

Among the undertakings of this period was the institution of a curriculum in engineering physics, with the emphasis on the physics, which turned out physicists prepared to enter industry. The success of this course proved that men trained in what we should now call industrial physics could enter industry at the bachelor's level, do valuable work, and advance in their profession. At the same time the department pushed forward its program of graduate work, and the more conventional undergraduate courses increased in interest and value as indicated by increased enrolment.

Some one has defined a dean as a man who has been so successful as a teacher that he is no longer allowed to teach. Whether or not this is valid, it seems probable that it was DODGE's success in building up graduate work in physics which led to his appointment as dean of the graduate school. Here he found larger opportunities to exercise his capacity for organization and cooperation. Thus he was able to promote better relations among all departments of the University, and also between the University and other educational institutions of the state. One of his important contributions to the state where he spent a quarter of a century was the organization and development of the University of Oklahoma Research Institute, which was so soundly planned that it immediately obtained wide support from industry. To physics as well as to other departments came numerous industrial fellowships that furnished great stimulus to graduate work.

I assume that you are all familiar with the important part which he played in the organization and early development of this association, of which he was the first president. But it may not be out of place to mention the extensive tour of the country which he made in 1932–33 as Field Director of the Survey of College and University Teaching of the American Association of University Professors. You are also familiar with his recent work as Director of the Office of Scientific Personnel, National Research Council, in which he not only helped to place trained specialists where they could best assist the war effort but also helped in the effective utilization of the limited teaching personnel left after the heavy drafts which war research made upon all physics departments. Some of us who had occasion to appeal for help to his organization are likely to believe that in this he made a contribution of no small magnitude to the teaching of physics in a difficult period.

Mr. President, it is my very great privilege and pleasure to present the selection of the Committee for the Oersted Medalist of 1944, DR. HOMER L. DODGE, teacher and organizer of teaching.

RESPONSE BY DOCTOR DODGE

THIS is the first time that the Oersted Medal has been awarded to a person whose contributions to teaching have been intimately associated with the work of the American Association of Physics Teachers. The Committee on Awards of our organization has refrained so long from anything that might partake of the flavor of mutual admiration that the accumulation of deserving individuals within our ranks is great. Perhaps the reason I was selected is that I happened to be the first president of the organization. I feel, therefore, that I am here largely as a symbol of the belief that the record of the Association shows 14 years of "notable contributions to the teaching of physics" and that any one of a number of persons who have helped make this history might just as well be standing here today. Thus I see in this award greater significance than a mere personal honor—though for this honor I say a deeply appreciative "Thank you."

It has been suggested that I say something about the early history and indicate the possible future course of the Association. The history that gets written down is often an emasculated, if not distorted, version of the facts, and it may be that you will find interest in some incidents in the history of the Association that might otherwise never see the printed page.

Just as the trunk of a tree springs from many roots, so any event has its origin in many causes. The tap root of the Association was the insistent interest in teaching that has characterized DOCTOR PAUL KLOPSTEG from the day I first knew him. It was in 1916, when he visited the State University of Iowa, that we talked at length and seriously and with youthful enthusiasm about the need for an active interest in professional aspects of physics teaching, paralleling the interest in research of the American Physical Society. Although there were sporadic

flurries of constructive discussion of this nature from time to time among physics teachers, it was he who kept alive through all the years the idea of an association. He talked about it to many people, and I crossed the path of the idea at a meeting in Atlantic City in 1930 in a discussion with a member of his organization. I recall as if it were yesterday that I sent back word to DOCTOR KLOPSTEG that if he would start the ball rolling I would push with all my might. He did just that, and with the help of DOCTORS STATES and WEBB assembled about 30 physicists interested in teaching in December 1930 at a luncheon meeting held at the time of the Cleveland meeting of the AAAS.

As chairman my chief contribution was to prevent the further postponement of effective action. I expressed the opinion that this was the best opportunity for organizing that we would ever have and that, if it were let pass, we might as well abandon the idea for a generation. The organization was effected, and the first annual meeting was held a year later, in December 1931.

At the organization meeting in 1930 some well-merited remarks were made about the neglect among college and university physicists of the problems of physics teaching, and some of those present were disposed to cultivate an aloofness toward other societies. I recall very well that PROFESSOR FREDERIC PALMER, who became the second president, took the position that we should work in the closest cooperation with the Physical Society. As a matter of fact, before the close of the meetings, I discussed with PROFESSOR F. K. RICHTMYER and PRESIDENT KARL COMPTON the great importance of having a generous overlapping of membership and of interest, and drafted them for service on the first executive committee so that it might be clear to all that the new association was to serve *physicists as teachers*, in the same way that the other existing societies had served them as researchers, and was not to degenerate into a society of pedagogs. The harmonious cooperation that has obtained through these 14 years is typified by the many joint programs, such as the present one.

At those same Cleveland meetings another physics organization was born—the American Institute of Physics—which was created to unify American physics. The need for special organizations in certain important branches of physics had led to a process of fission, which needed to be counteracted by a cohesive influence. Some of you will recall that there was at first some question as to whether the Association had matured sufficiently to be recognized as a founder society of the Institute. That the favorable decision was a wise one is evidenced by the fact that the present Chairman of the Institute's Governing Board is one of the Association's representatives on that Board and that various officers and members of the Association have worked effectively and faithfully on a number of important Institute projects.

It has been natural already to mention several names in this brief recital of Association history. I could go on and mention all the officers, past and present, but still would not have covered all who have made great contributions to the Association and to teaching. Better to stop now with each of you thinking of those men who, as I have said, might stand here today as well as I.

Mention should be made, however, of one more event whose significance has increased with the years. I refer to the establishment in 1933 of the journal, first called *The American Physics Teacher*, and now the AMERICAN JOURNAL OF PHYSICS. Perhaps it is well that I had had previous experience as a presiding officer of faculty meetings and understood how a chairman, under stress of necessity, can influence action. Knowing that the very life of the Association depended upon having a journal and finding that the membership was not quite ready for the decision, the chairman had to assist the birth by some parliamentary procedures that would not be found in Roberts' *Rules of Order*. As we all know, the Journal owes much of its success to the superb leadership of PROFESSOR DUANE ROLLER, who has brought to the work great editorial skill, a broad vision of the significance of the teaching profession, and a devotion to the job that is rarely equaled.

One is tempted to carry the history of the Association beyond the early years and to discuss its ever-enlarging program made possible by a membership increasing not only in numbers but also in its devotion to the ideals and activities of the Association. During all these years we have stood, above all things, for a broad vision of the functions of a physicist in his profession and in his relationships to other fields of learning, to industry and to society in general. We believe that the research physicist is a teacher more often than he realizes and that research will prosper better if research physicists will take a greater interest in the teaching of physics at all levels. In fact, although we grant that the relative emphasis on research and teaching depends on

the individual and varies accordingly, we believe that the two are not only compatible but go well together. We see no place for the research man who looks down on teaching, for we know that such an attitude indicates narrowness of view on his part and a complete lack of appreciation of what teaching means. Likewise we want no teacher of physics to be unappreciative of the fact that the fountainhead of scientific progress is in research and that one of the chief functions of teaching is adequately to prepare for a research career those whose talents lie in that direction.

Reference has been made to the unifying influence of the American Institute of Physics. Some of the inspiration for the constructive and, may I say, imaginative approach to the problem of developing the spirit of unity in the physics profession has undoubtedly had its origins in the type of thinking that has characterized the leaders of the Association.

We have always believed that, by the very nature of their subject, physicists cannot possibly avoid, and should not seek to avoid, participating in all phases of activity of our profession and exerting influences in those overlapping fields where our assistance and guidance are needed, especially in engineering. The Association holds fully and frankly to the proposition that we are not separately "industrial physicists," "research physicists," "physics teachers," and so on, but that *physicist* is and should be an all-inclusive term embracing every trained person whose primary concern is with some phase or combination of phases of the science. These phases may include research in pure and applied physics, teaching of physics, developmental research in industry, and work in such borderline fields as biophysics, geophysics, and that wide range of engineering applications which may collectively be called "engineering physics."

Our Association is made up of those physicists who, because of particular concern with the teaching phases of the science, are destined to have in their classes the oncoming generations of physicists. To us is given the opportunity not only to teach our subject to the prospective physicist but also to open up to him and to the general student its wider vistas leading to general recognition of the fact that a person is not educated unless he has some genuine understanding of physical science. May the future conduct of the Association express our belief in the unity of our profession and richness of our science.

Meeting of the Oregon Chapter

The third annual meeting of the Oregon chapter of the American Association of Physics Teachers was held at the Public Library, Portland, on January 13, 1945, in conjunction with the Oregon Academy of Science. The following program was presented.

Physics buildings in American universities. WILL V. NORRIS, *University of Oregon*.

Controlling the properties of photographic emulsions. J. C. GARMAN, *Oregon State College*.

D'Arsonval galvanometer analysis. W. R. VARNER, *Oregon State College*.

Computations: theory and practise. E. L. PETERSON, *University of Oregon*.

Remarks on the training of physicists. R. R. DEMPSTER, *Oregon State College*, and D. S. DEDRICK, *Development Laboratory, Weyerhaeuser Company*.

It was voted that a committee, consisting of A. A. Knowlton, Brother Godfrey, D. S. Dedrick, W. Haynes, A. A. Groening and H. Esterly, be appointed to investigate industrial cooperation with physics departments of colleges and schools in Oregon. It was also voted that the Oregon chapter affiliate with the Oregon Academy of Science as an organization.

E. HOBART COLLINS, *Secretary*

DESIGN OF INSTRUCTIONAL LABORATORIES: A SYMPOSIUM

NO phase of the art of teaching is more dependent on properly designed and appropriately furnished buildings than is instruction in the laboratory sciences. Yet there is an astonishing dearth of collated reference material on the subject. One who incurs the inspiring yet highly exacting responsibility of supervising the design and construction of a new laboratory can find little to help him in scientific literature except the descriptions of a few completed laboratories. These are of course helpful, as far as they go. But the visible end product gives about as much information on the problems involved in attaining it as the photograph of a baby gives on the preceding problems of conception, pregnancy, and parturition.

With a view to giving aid to teachers of physics in this respect, a symposium on "Design of Instructional Laboratories" was arranged for the January 1945 meeting of the American Association of Physics Teachers. The original plan was to subdivide the subject into the various phases of the building problem and, for each paper, to present a digest of the best practice (within each phase) among all recently constructed instructional laboratories. Actually the papers were rather largely limited each to its own account of a particular building. In at least some of the cases, however, the practice established in these particular buildings was so far in advance of the general level elsewhere as to constitute some justification for such a limitation.

It was intended to confine the topics to those specific to physics laboratories, and this was in the main done. The treatment was by no means exhaustive, however, even within this fairly narrow range. The four topics of the symposium, with the speakers, were:

The functions of the housing committee. W. P. DAVEY.
Floor plans. C. J. OVERBECK.
The lecture room and its equipment. J. W. BUCHTA.
Laboratory electric services. C. E. HOWE.

Though the series can scarcely be said to fill the serious lacuna in physics literature which was the occasion of its initiation, it is a substantial beginning. Perhaps readers who have had significant building experience may be moved to add other topics, or at least to give the profession the benefit of their experience on important details through letters to the Editor.—LLOYD W. TAYLOR.

The Functions of the Housing Committee

WHEELER P. DAVEY
The Pennsylvania State College, State College, Pennsylvania

THE first step in planning a new physics building should be the appointment of a "housing committee." This committee should preferably be composed of three men. In the case of a large department all three should be members of the physics staff, or two should belong to the physics department and one to the chemistry department. In the case of a small department which may have to share its building for a few years with biology and chemistry, each department should be represented on the committee. At least one member of the committee should be able to handle drawing tools. At least one member should be able to visualize a room in three dimensions when looking at a floor plan and the corresponding four wall elevations, and should have some little knowledge of the building trades and of the state safety laws. If such a person does not happen to be available in the physics department, one should be brought in from some other physics department as a consultant.

The housing committee should be granted in writing full authority to make all final decisions in regard to sizes, shapes and general layout of rooms, and, when necessary to the successful functioning of the rooms, to decide the locations and grouping of rooms. The committee should be empowered, too, to request data and specific information from members of the departmental staff, and to set a time limit for the delivery to the committee of such data and information. Unless the college administration is willing to delegate authority in all the foregoing matters, it should not appoint a housing committee.

The first action of the housing committee should be to make a reasonable estimate of the growth of the physics department. To this end, the president of the college should be asked for his estimate of the probable over-all growth of the college during the next 20 years. The committee should multiply this figure by a factor representing its own estimate of the probable change in the relative importance of physics as compared with the other subjects taught on the campus. This change in relative position will reflect the attitude and vision of the departmental staff, the financial support of the department, the type of college, its location, its sponsorship and the basic

policies of its college administration. These estimates, together with a knowledge of the details of the prewar enrolment of students in the various physics courses, will enable the housing committee to set a percentage expansion which must be taken into account in planning a new physics building and in providing for its proper equipment. The committee should then tabulate the estimated number of students and of student-hours for each physics course that is now offered or that would probably be offered after the department has attained its estimated growth.

The second action of the housing committee should be to find out exactly what the members of the physics department feel they need in order to do their work as well as possible. Every person in charge of any of the activities of the department, including not only the faculty but also the instrument maker, the stock man, the librarian and the superintendent of grounds and buildings[1] should be included in the request for information. Each instructor in charge of a course should be handed a statement giving the prewar enrolment in that course and the enrolment anticipated by the committee. He should be asked to submit to the committee by a certain date a set of pencil drawings to scale ($\frac{1}{4}$ in. = 1 ft), showing the floor plan and all wall elevations of the space which he estimates will be needed for the course if the enrolment grows to that predicted by the committee. These drawings should go into considerable detail. For instance, classroom plans should show (i) the location and spacing for every seat (including the number and location of seats for left-handed students); (ii) the location and size of the instructor's tables; (iii) the location of all blackboards, chart-hangers, etc.; (iv) the location of electric outlets for projection lanterns, clocks, etc. Similar drawings for each lecture room should indicate, in addition to the foregoing, the location and size of aisles, (note legal requirements), projection booth, lecture table and its services (a.c., d.c., water, sink, air, gas, etc.), the size and design of the lecture-room, stockroom, etc. The drawings for instructional laboratories should show to scale the size and location of every table, shelf, cupboard, sink and drain, and the location of every gas cock, air cock, a.c. and d.c. outlet, etc.

[1] The superintendent of grounds and buildings will be interested not only because of the space assigned to him for heating and ventilation control and for janitor service but also because of the increased demand for water, steam and electric power due the new building. In case his power plant is already running close to the limit of its capacity, he may even need to expand his facilities before the new physics building can be occupied.

The instrument maker should be asked to plan a shop adequate for the estimated size of the department, showing to scale the size and location of every machine (at least a lathe, shaper, miller, grinder, band saw and power hack-saw), work bench, assembly space, etc., all with proper aisles designed from a safety standpoint. Drawings should be requested similarly from the departmental librarian, the stockman and the superintendent of grounds and buildings.

As soon as all these drawings are available, the housing committee should study them in detail. For instance, three circular disks, respectively $\frac{3}{4}$, 1 and $1\frac{1}{4}$ in. in diameter, should be cut out of heavy paper. The $\frac{3}{4}$-in. disk represents a man walking; the 1-in. disk, a student in a classroom aisle, or a man bending over to get some apparatus from a cupboard which has been built below a table; the $1\frac{1}{4}$ in. disk, two students working back to back in an aisle that lies between two laboratory tables. Every aisle and every passageway should be tested with the appropriate disk, to make sure that all clearances are big enough to be safe. The committee should then examine the floor plans for each room to determine whether or not the amount of space asked for is reasonable. In the rare cases where too much or too little space has been requested, the committee should make the necessary adjustments. In the light of the various floor plans submitted, the committee should select for each grouping of rooms a standard width and a standard unit of length such that the rooms in that group can be made exactly 1, 2 or 3 units long. The committee should then request that all floor plans be revised to conform to that width of room and to the unit adopted for length of rooms. The next step is to check any possible overlapping of space requirements, for example, to see if one classroom can be reasonably expected to take care of the needs of two teachers, each of whom has asked for a room for the course of which he is in charge. Having finally decided upon the number, shape and sizes of the rooms, the housing committee should add hallways, offices, stairs and toilets (note state and local safety regulations), and should assemble the whole into floor plans for the various floors of the proposed new building, making sure that rooms are grouped together in such a way as to enhance their usefulness. These floor plans are not to be considered as final building plans. They are "space-functional drawings," intended to express in architect's language the shapes, sizes and

grouping of rooms so that they may be as useful as possible. Notations should be made on these drawings of any special requirements such as type of artificial lighting, acoustic plaster, artificial ventilation, ceiling heights and floor loads.

The housing committee is now ready to confer with the architect, for it now has all the information that he will need for his preliminary building plans. The committee should at all times observe the proper division of labor and responsibility between its own work and that of the architect. The committee should never attempt to locate windows or doors except where such location is absolutely necessary to the proper functioning of the room. In general, the committee should have nothing to do directly with the exterior appearance of the building, structural details, the presence or absence of an ornate lobby, etc. Such considerations ordinarily fall entirely within the province of the architect.

In order that the exterior of the building may conform to his ideal of beauty or in order, say, to insert some extra stairway required by law, the architect may find it necessary to regroup some or all of the rooms or to move some of the room partitions. In such a case, he should confer with the housing committee to make sure that his proposed changes will not impair the proper functioning of the rooms. The committee will find the original proposals for wall elevations invaluable in its conferences with the architect. As soon as committee and architect reach an agreement, the latter should furnish a complete set of floor plans. The committee should at once provide each person who originally contributed plans with a set of drawings to scale showing the floor plan and wall elevations that have been finally decided upon for his room or rooms, and should request that he draw in to scale, with pencil, his final recommendation for the location of shelves, tables, cupboards and service outlets. After these pencil drawings have been checked, they should be traced on tracing cloth and copies sent to the architect as functional drawings for his use in making final "detail drawings." With the delivery of these drawings to the architect the housing committee has completed that portion of its duties which has to do with the design of the building.

There remains the matter of furniture and equipment. The superintendent of grounds and buildings and the purchasing agent will be helpful in choosing manufacturers of suitable classroom seats, instructor's classroom tables and office

Form I. List of apparatus needed.

Name of course..... Room No. in new bldg..... Instructor's name.....

No. on hand	No. to be ordered	Total	Name of item	Mfgr.	Catalog No.	Unit cost	Total cost
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...

furniture. These men should be consulted freely, but in most colleges the final choice should rest with the housing committee. In the matter of manufacturers of shop machinery and tools, the committee should confer with the departmental mechanic and with the purchasing agent.

The selection of additional laboratory apparatus is much more complicated. The housing committee should request each member of the physics staff who is in charge of laboratory classes to furnish for such courses, on a form supplied by the committee (see Form I), a complete list of apparatus needed for the proper operation of the course.[2] The committee should then transfer each item to an individual library file card so as to permit sorting according to function, preferred manufacturer, etc. as occasion may require. If apparatus must be purchased on the basis of competitive bids, the committee must make a set of specifications for each item (Form II).

The committee will be visited by many salesmen of machinery, equipment and apparatus. Most of these men will be found quite helpful, a few will be plain pests. The committee should have as an inflexible rule that all statements by salesmen must be submitted in writing and signed with the salesman's name and the name of his company. All oral statements such as "just as good" or "better than" should be disregarded as not being binding. The committee should insist upon the right to reject bids *for individual items* either on the basis of cost or on the basis of

Form II. Specifications for each item of apparatus.

Name of item.....; number needed.....
This piece of apparatus or equipment must conform in general design, dimensions, materials of construction, workmanship, finish, resistance to corrosion, limit of accuracy, and ability to coordinate with existing apparatus and equipment now in use at.....
to the item manufactured by.....of.....
under its catalog designation of....., No.....

[2] In this connection he should refer to "Criterions for self-rating of physics departments," Am. J. Phy. **12**, 353 (1944).

failure to conform to all the restrictions in the specifications.

After the successful bidder for a given item has been determined, his name and the amount of his bid should be entered on the back of the afore-mentioned file card. A separate record should be kept of the names of unsuccessful bidders and the exact reasons for rejecting their bids for each individual item. All this obviously entails a considerable amount of bookkeeping, but will be found to be decidedly worth while. Delivery of apparatus should be made to the committee, and the committee or its representative from the physics staff should examine each piece to make sure that it arrives in proper condition. The date of receipt of each item and the approval of the committee's inspector should be recorded on the back of the file card so that, merely by sorting the cards, one can know what items are still "back-ordered."

Having completed its work on the functional requirements of the new building and on the necessary equipment for the successful operation of the building, the committee should request that it be discharged and that all future matters of operation of the building be taken care of through the usual and regular channels. Such action on the part of a housing committee that has finished its work may save its members considerable embarrassment at some later date.

Floor Plans

C. J. OVERBECK
Northwestern University, Evanston, Illinois

MANY factors must be taken into consideration in the process of producing the best possible floor plans for a highly specialized building such as a science laboratory. Some of the factors, and these may be most important, are entirely local, such as the size of the department and number of students served, types of research advanced, expected expansion, funds available, topography of site, general style of architecture to be maintained, proximity to other buildings, and location with respect to thoroughfares or other sources of noise and vibrations. Thus we can agree at the beginning that there is no such thing as a "standard" floor plan that can be adopted without modification by all institutions.

There are, on the other hand, certain general characteristics of all well-planned science buildings. These characteristics as presented here are based on a study of available literature, of which there is a surprisingly limited amount, and on personal contact with college laboratories. The primary purpose of this study is to call attention to features that must be considered in building a new laboratory and to show how various institutions have handled them.

Many of the problems are simplified by the careful selection of a suitable site. One of the major considerations is freedom from mechanical disturbances and noise. This means separation from large thoroughfares and from buildings in which heavy machinery, especially reciprocating machinery, is operated. In most cases a reasonable expansion of the department should be anticipated. To avoid growing like Topsy, it is wise to consider the suitability of the site both for its immediate use and for a possible extension later. The building should be set so that the major student traffic flows naturally past its main entrance.

The general architectural features likely will have to conform to those already adopted for other campus buildings. Insofar as possible, structural stability, interior adaptability and simplicity should be stressed in the floor plans. The major load of the building should be supported by strong, heavy exterior walls, supplemented, if necessary, by a few interior walls. A reinforced-concrete framework and brick or stone are most commonly used. Since there is no way of accurately determining all future needs, free interior walls, even though initially more expensive, may in the long run be most economical. Interior changes may then be made as time dictates new arrangements. Plaster on hollow tile is common for these free walls. In many buildings the corridor walls are double, with the clear space housing the conduits, water pipes, and ventilating and air-conditioning ducts. Some means should be provided for readily supplementing these electric lines and other services. It is a wise practice, however, to incorporate in the original construction, lines of larger capacity than immediate needs dictate and even some that are initially unused. Spare space may be utilized by

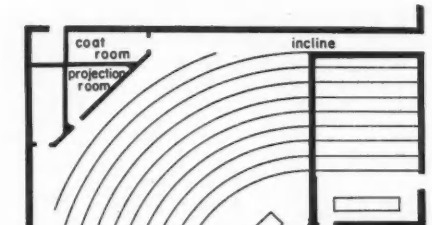


FIG. 1. Lecture rooms.

recessing portions of one wall for storage cabinets, lockers, and display and teaching cases.

Simplicity of design, which normally involves symmetry, can usually be expected to improve working conditions. The main entrance and staircases should be centralized to minimize student travel and confusion. Long straight corridors also reduce commotion and facilitate rapid general and emergency exit. A double escape system should be provided for all passageways to meet emergency conditions. A corridor of width 8 ft will accomodate the normal traffic load.

A uniform illumination of 25 or 30 ft-ca at the working area is excellent. Acoustic treatment of the ceilings, especially in the corridors, large lecture room, and elementary laboratories is money well spent and almost imperative for large institutions. Air conditioning or some other means of controlled ventilation should be provided in the lecture and class rooms. The extent of its use in other parts of the building is usually a matter of the funds available.

The subdivision of the building into sections to handle the different duties of a science department must, of course, be in terms of securing a maximum of desired characteristics for each with a minimum of mutual interference. The first-year work, which involves the largest number of students, should be concentrated on a lower floor, preferably the first, near the main entrance. This will again help to minimize traffic congestion, and the inevitable commotions and vibrations will then interfere less with the functions of other parts of the building. Of the three types of rooms needed—lecture, class and laboratory—the laboratories are least disturbed by a noisy location.

Lecture rooms will be discussed in detail in a subsequent paper. Suffice it to say here that a capacity of about 200 students is near the upper limit for suitable visibility of demonstration experiments. In the case of large institutions several lecture rooms of varying size may be so placed that a single large apparatus and preparation room will serve all of them. Figure 1 shows one possible arrangement. Here the large room has a capacity of 200 students, and the smaller rooms, about 50 students each.

Classrooms for physics are similar to those for mathematics in that extensive blackboard space is needed. In the belief that student participation in class is very important, the capacity should be limited to 30 students per room or even fewer. The type of room shown in Fig. 2, of floor area about 420 ft^2, has been found very satisfactory. It contains 28 fixed table-arm chairs spaced for free student movement. The blackboards b of roughened black glass, 4×34 ft, extend along the front and side of the room. The back row of seats, sufficiently far from the rear wall, has behind it a line of coat hooks h. Many classrooms are scenes of disorder with wraps dumped on the floor or occupying seats, because that detail was forgotten. The identical rooms along the corridor have their exit doors staggered, as shown, to reduce congestion when classes are dismissed.

The three types of laboratory rooms—elementary, advanced and research—require separate discussion. Each will be considered in turn, after certain general features, common to all, have been discussed. The plans must provide for:

(a) Ample services located in positions most suitable for their use. Electric services (the topic of another paper in this series), gas and water, with an acid-resisting sink or small drain, are the minimum requirements. Steam, compressed air and vacuum lines are installed in some of the more recently built laboratories.

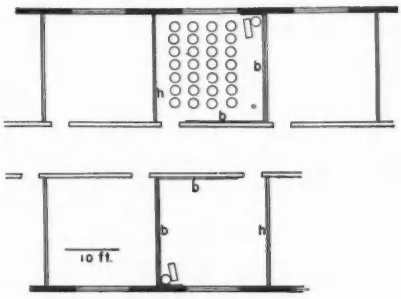


FIG. 2. Classrooms.

(*b*) Strong, durable work tables with acid-proof tops of edge-grain maple or other hard wood. It is an advantage to have a very wide overhang on these for holding the table edge clamps. With modern construction, which limits vibrational disturbances, the individually mounted masonry pier is now seldom required for experimental work.

(*c*) A means of readily attaching wall equipment without marring the walls. For this purpose iron wall channels or heavy wall plank (2×6 in.) should be rigidly mounted horizontally on all laboratory walls at heights of about 3 and 6 ft. Several buildings are equipped with spaced wall and ceiling inserts which are threaded for attaching equipment.

(*d*) Darkening facilities. Lightproof shades are essential to most laboratories. A combination of shades and Venetian blinds will help provide ideal working conditions. At least one university reports using a light-weight fiber door, placed within the window frame and opening inward, for darkening its optics laboratory.

(*e*) Storage facilities for equipment. For the student laboratories the allotted space is frequently underestimated. A small cabinet is sufficient for the individual research rooms.

A study of articles on elementary physics laboratories reveals the general opinion that a laboratory class of from 16 to 20 persons under one instructor is most satisfactory. A room with 1000 ft² of floor space can accommodate this number without crowding. The rectangular room of Fig. 3 has ten tables (3×6 ft) and a wall bench 27 in. wide along one length. Wall equipment space is provided along the other length and the back of the room. Near the front of the room is a coat rack *h*, a record and report cabinet *c*, and a blackboard *b*. The main switch board *s* controls all electric power in the room and feeds to the small power panels *x* for wall equipment and to the "power islands" *p* which serve the tables.

These power islands *p* are permanently placed. Each island has duplicate units of all services—a.c., d.c., battery, time impulse lines, and gas outlets at the front. There is a sink with hot and cold water which is covered by a folding top when not in use. Thus each pair of tables has a nearly continuous 3×14-ft surface, which is advantageous for certain experiments, for example, those on vibrating strings. These fixed power islands permit the use of movable tables if desired. No inconvenience has been occasioned, however, in this laboratory with fixed tables,

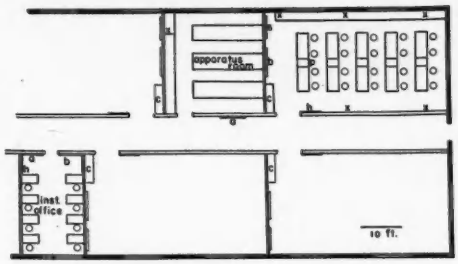


FIG. 3. Elementary laboratories.

which have also helped to create an air of orderliness.

An instructor's desk has purposely been omitted. Its absence sometimes improves the attention given to the students. The doors of the room are unusually wide, to facilitate the shifting of apparatus and permit easy passage of the rubber-tired apparatus truck which is a time-saver. In institutions serving large classes the duplicate laboratory rooms should be adjacent, as shown in blank in Fig. 3. The young and the new instructors, usually the ones assigned to this work, will find their tasks made easier by complete duplication in the rooms, especially of electric services.

Adjacent to, or centered in, these laboratory groups, is the proper place for the apparatus room. This apparatus room should have suitable cabinets containing adjustable, well-lighted shelves to accommodate the varied apparatus. A small, suitably equipped work bench for making minor adjustments may be appropriately added. This room should be of adequate size; 600 to 800 ft² of floor space will house the usual equipment for a ten-group parallel-front laboratory system.

A room in this section may well be equipped with desks and serve as an office for the laboratory instructors when several instructors are required. It provides a central place for conferences between the student and his laboratory instructor. The person in general charge will find that he can more efficiently and effectively supervise and correlate the work of the several courses through this office. The hall bulletin board *a* should be the official means of getting assignments and information to the students. It is available even when the laboratories are closed.

The advanced laboratories, usually serving a limited number of students, should be much more flexible than the elementary. The space allotment per student is normally much larger than for the first course. More attention must be

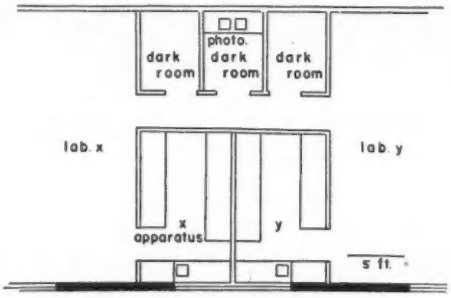


FIG. 4. Advanced laboratories.

given to vibration isolation since a higher accuracy is maintained. Space must be provided for apparatus storage and for minor repair work. In courses such as optics and electricity, darkrooms are required. Figure 4 suggests a possible plan of conveniently sharing such rooms. Some of the existing plans add a small, equipped lecture table and a blackboard to these rooms so that they may also be used as classrooms.

The general research rooms should be located in sections of the building isolated from undergraduate student activity and sources of other disturbances such as the shop and compressor rooms. Therefore, research rooms are usually placed on the top floor and in the basement of the building. A check of available plans indicates that a floor space of from 250 to 350 ft^2 is commonly set aside for each individual research room in the science buildings recently constructed. This size can accommodate, besides the sink and service units, two work tables and a small apparatus cabinet. Generally a length of the floor is subdivided into a series of adjacent research rooms. It is a good practise to provide sleeves and cover plates through the walls near the ceiling and the floor of such rooms to permit the passage of temporary services from one room to another. Some means of securing such flexibility is highly desirable.

Another scheme, used especially in industrial laboratories, is to congregate a number of projects in a single field, for instance, spectroscopy, in one large room. Certain expensive units of equipment can thus serve several groups. As one report on this plan points out, "each man benefits from the experience of his fellows in the large room, and all have the possibility of constant advice and inspiration from their thesis supervisor, whose headquarters are in the same large room."

Specialized research rooms cannot adequately be discussed here. Their unique problems of temperature and humidity control, electric and x-ray shielding, sound and vibration damping, and so forth, are topics that require extensive discussion. It may be desirable to provide a two-story room, or two rooms separated by a removable floor, for large-scale projects, even though this is not immediately needed.

The shop, generator and compressor rooms, which are sources of vibration and noise, should be in an outlying section of the building, removed as far as possible from the research section. They should be on the ground floor and near an entrance and the service elevator. The floor plans to allocate space for the shop machines required depend on the size and activity of the department. For small institutions the general wood- and metal-working tools necessary for general repairs may have to suffice. If the activity justifies trained instrument makers and a fairly complete shop, the plans should include also a separate shop for graduate students and staff. This is a protection for the fine main shop machines. Connection with a well-equipped engineering department reduces the need for heavy shop machinery. Adjacent receiving, stock and storage rooms eliminate the necessity of cluttering the shop with extra supplies. For the same reason, a "morgue" should be included to house the obsolete equipment of the department—parts of which may later be needed for other purposes. All shop machinery, power generators and other similar equipment should be mounted on vibration-absorbing bases. For the very large units, such as air compressors and liquid air machines, a separate base, independent of the building, is desirable.

A floor space of about 175 ft^2 is sufficient for an individual office for each major staff member. This will accommodate a desk, work table, file case, bookcase, coat stand and chairs. Exterior windows obviously are desirable. A small blackboard (4×6 ft) is convenient for individual student conferences or help sessions. Whether or not all staff members have offices adjacent to a central departmental office is a matter of diverse opinion as evidenced by the plans selected for existing laboratory buildings. There are good arguments for such centralization in correlating the work of the department and establishing a central point for student conferences. The strongest argument against this arrangement is that it prohibits locating the office and research activity of the instructor in adjoining rooms of the building.

The departmental library and a small room for informal meetings of the staff and graduate students should be situated near the main office. The nature of the work of physics requires an unusual degree of cooperation of the staff. Many such departmental conferences have been helped by the addition of a hot-plate in a nook off this room.

This paper will have served its purpose if it has brought to your attention details which might otherwise have been forgotten in the delightful task of making floor plans for your new laboratory.

The Physics Lecture Room and Its Equipment

J. W. Buchta
University of Minnesota, Minneapolis, Minnesota

IN preparing notes on what should be said about the physics lecture room and its equipment one is disturbed by the fact that what he has to say should be obvious and well known. Yet as he looks about him in his own lecture room and as he visits other laboratories he finds that many of the "obvious" things have been overlooked. I shall, therefore, list both the obvious and those items that may not be so classified.

If the floor plan of the lecture room has the proper form and the floor slopes toward the lecture table, the room may be large enough to accommodate as many as 250 persons and still allow the audience to see lecture demonstrations fairly clearly. The floor plan of a room for this number of people should be approximately square. Long, narrow rooms for large groups are not satisfactory for showing demonstrations. Of course, the room may be made larger if it is to accommodate classes other than those to whom demonstration lectures are to be given. Supporting posts in a lecture room should be avoided and usually can be. Steel supporting beams are not excessively expensive and, if one insists, the architect will usually comply with a request regarding supporting posts.

General floor plans and the size, form and the placing of lecture rooms have been discussed by Professor Overbeck in the preceding article. He has indicated the desirability of having the apparatus rooms and lecture rooms centrally located on the first floor and has shown such a plan. I shall discuss a second plan (Fig. 1) and point out some of its desirable and undesirable features.

The three lecture rooms illustrated in Fig. 1 are directly supplied by one lecture apparatus room. This room has no outside windows, and precautions are taken to keep it reasonably free from dust. Ventilation is supplied through the doors of the lecture rooms and a vertical shaft to the roof, which may also be used to bring in a beam of sunlight. Relatively wide doors are used for the apparatus room to facilitate moving equipment in and out and, since the room is at a lower level than the door to the entrance hall, a ramp with reasonable slope is used. This permits the use of a truck which could not be used on a stairway. The apparatus room has a workbench, a small assortment of tools, a sink and the cabinets for apparatus. These cabinets have adjustable shelves, each shelf with its own light.

The lecture room entrances are placed near outside doors. The audience need not go through long halls nor up and down stairs to enter any of the lecture rooms.

Each room is nearly square except the largest. This will seat 500 people and is also used for other than physics classes. Physics classes of 250 can be seated so that they can see the demonstrations on the lecture table. The rear corners of the room, which are least useful for an audience, are blocked out for use as storerooms.

No provision was made in these rooms for coats. In Minnesota such provision is desirable.

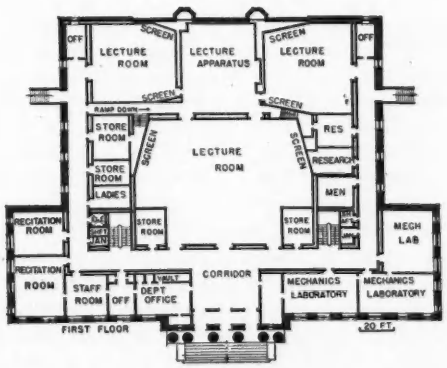


FIG. 1. A floor plan showing arrangement of apparatus room and lecture rooms.

In some newer buildings on our campus, rods are placed in recesses in the walls. These seem preferable to separate coat rooms. The coats are always in sight, there is less crowding at the beginning and end of an hour, and less time is required to hang coats and to obtain them than when a regular cloakroom is used.

The arrangement for projection is somewhat unique in the afore-described room. Above the blackboard in the front of each room there is a permanent white plaster screen. This is used chiefly when motion pictures are projected by a portable machine at the back of the room. But for the majority of projections during regular lecture hours we use the permanent screen surfaces on the side walls. These walls are set at an angle of about 25° to the main part of the side walls. The angles were chosen partly because of structural considerations. They are also a satisfactory compromise between distortion and ability to see the screen from seats near the side wall. In the larger room aisles are placed next to the side walls so that there are very few of an audience filling the room who cannot see the screen satisfactorily. These aisles near the wall are also desirable for quick clearance of the room, passing out papers, proctoring, and so forth.

The arrangement of screens on side walls permits the lecturer to operate his own slide lantern. From experience we know it to be much more satisfactory than arrangements which necessitate a lantern operator or require a lecturer to leave his place behind the table.

Since there are two screens set at an angle, one at each side of the lecture room, we have experimented with a half-silvered mirror. With an ordinary lantern and a half-silvered mirror one can project images on both screens simultaneously. This, of course, gives a mirror image on one of them, but this can be corrected by a reversing prism. Seldom, however, is the room so crowded that use must be made of the seats adjacent to the wall on which the screen is placed; and, unless these seats are used, all can see satisfactorily.

Side screens permit the lecturer to show by shadow projection many experiments that would otherwise be difficult or impossible for the audience to see. A small arc as a light source will give good shadows of charged pith balls or the leaves of an electroscope. More unusual experiments, such as those on the magnetic properties of liquid oxygen, can also be shown. A boiling bridge of liquid oxygen hanging between the poles of an excited electromagnet and falling when the current is turned off can be shown by shadows to several hundred students.

With a large condenser lens in front of an arc and a projection lens of proper focal length at a distance from the condensing lens, one has essentially a large projection lantern in which small objects can be placed and their images shown on a screen. Small pieces of apparatus—for example, the parts of a small radio tube—can be pointed out and described to a large class. *Demonstration Experiments in Physics*[1] describes a number of experiments in which the arrangement is useful. The screens on the side walls are particularly desirable for such demonstrations.

The large lecture room (Fig. 1) is apparently not provided with a projection booth. Actually the room on the side marked "research" is arranged for projection but has never been so used. We do not have a permanently mounted motion picture machine, and portable machines are set up in the lecture room itself. If a projection room is provided for the possible later installation of a permanently mounted machine, one should remember that it requires more than the ordinary electric circuits. Fire protection must also be considered.

In older lecture rooms skylights or outside windows were usually provided. In most modern rooms they are not necessary. The mechanisms for closing and opening the shutters over the skylights and for raising or lowering the window shades usually cause so much trouble that they are not used. Omission of the skylight often reduces heating costs. With present-day artificial lighting and air conditioning, skylights and windows can be omitted.

The lecturer should be able to control the lights of the room without leaving his place behind the lecture table. By means of relays and a master switch the lights can be arranged so that all or a part of them can be controlled from several positions along the lecture table, or a cord can be placed along the back edge of the table so that they can be controlled from any position. The lecturer should not have to hunt in the dark for the switches. Small neon lights are useful in locating switches.

In some lecture rooms, as in the new building at Northwestern University, provision is made for controlling the light intensity from zero to full value. This would make it a little more complicated to arrange for control at any point along the lecture table, but a physicist should

[1] R. M. Sutton, ed. (McGraw-Hill, 1938).

surely be able to meet the situation. In fact, one sometimes wonders why the physicist does not make more use of the various gadgets at his command in the design of lecture rooms and laboratories. For our own lecture rooms with screens on the side walls I have been tempted to put photoelectric cells in small recesses in the screen arranged so that when the light from the projection lantern or arc falls on the cells the house lights are turned off and then are turned on when the projection lantern is extinguished. When priorities are lifted we may try such a gadget.

Lecture tables and blackboards should have adequate lighting controlled separately or jointly with the other lights. Lights placed in reflectors above the blackboards and lights below the edge of the lecture table and directed toward the blackboards are satisfactory for its illumination. The lecture table will be adequately lighted by a group of modern sealed spot lights placed in recesses in the ceiling.

We have found that ordinary white or ivory floor linoleum is a good lecture table covering. The material wears well and gives a better background for showing apparatus than the usual black top. Some of the new plastic materials available in a light color may serve even better than linoleum.

The lecture table should have gas, air and electric outlets distributed along its length. In this way apparatus can be set up at the most convenient place and need not be restricted to one end. A large-volume air line in addition to the small size is useful. Outlets for large currents, polarized arc-lamp outlets, three-phase a.c. connections, house battery terminals, and a good ground connection as well as usual house outlets for a.c. and d.c. should be available along the table.

A galvanometer, or two of them, placed at the rear of the room with optical arrangements to project spots of light on the front wall and screen are very useful. Several connections of the plug and jack type for the galvanometer circuits should be placed along the rear panel of the lecture table. A control box which permits adjustment of galvanometer sensitivity may be placed in a drawer of the table. One may also use a galvanometer on the table or a wall bracket on the front wall and arrange an optical system to project the beam on a screen above the blackboard.

A sink, and hot- and cold-water faucets should be placed at the end of the table or in the center. A water-proof floor and a floor drain are desirable. Experiments in which water is spilled may be performed there.

Some lecture tables are made with movable sections. There seems to be some disagreement among teachers as to their practicability. A movable center section would have some advantages in performing certain experiments. Lecture tables may have drawers at the ends, but for the most part they should be open at the rear.

In place of movable sections of the lecture table we have used trucks which are of different sizes and shapes but have the same height as the lecture table. One having the same width as the table can be wheeled up to the end of the lecture table. A long, narrow one can be placed at the front or back of the table and still not disturb the lecturer. These trucks carry electric outlets and can be connected to the circuits of the stationary table. Such movable tables or trucks permit one to assemble and adjust apparatus before it is placed in the lecture room and are particularly useful when the lecture room is used for consecutive class hours.

An opening in the floor behind the lecture table and into a suitable space in the room below is useful for such experiments as the conduction of sound through a long rod. Means should also be provided to hang apparatus from the ceiling above the table. If there is space above the ceiling in which one can work, ports may be placed in the ceiling for use in hanging apparatus. If such space above the ceiling is not available, a track may be hung from the ceiling and above the table. A ladder may be necessary for hanging things from this track. If the ceiling is not high, a shaft of some height above the ceiling should be constructed in which such devices as a Foucault pendulum can be hung.

Two or three channel irons placed parallel in the front wall of the room will permit one temporarily to hang apparatus on the wall without defacing the wall.

A multiple vertical, sliding blackboard has many features in its favor if it is well constructed and does not rattle when used. A fixed surface and two other boards which may slide upward and under the projection screen on the front wall should be adequate for most purposes. The movable boards may be balanced on weights or they may be motor driven with controls placed along the lower edge of the board.

Certainly a room in which the principles

underlying the proper acoustics of auditoriums are taught should itself have good acoustic characteristics. In this part of the planning we have found it desirable to check the calculations of the architect. He may decide on the introduction of some hard smooth plaster for decorative purposes and believe that a little less of the acoustic material and more of the hard plaster will make no difference. Today there is no excuse for an acoustically poor auditorium.

Ventilation fans that serve each lecture room individually and that can be controlled from the lecture room are preferable to a system in which the lecture room is dependent upon fans that serve the whole building. Ventilation systems should be quiet and can be made so if specifications for it are included in the contract. Some systems that I know of must be turned off when a lecture is being given because of the noise level of the system.

Bulletin boards are desirable; they should be outside the rooms and far enough from the lecture-room entrance doors to avoid crowding.

The most important item of all lecture-room equipment I have not mentioned. That is an enthusiastic and imaginative lecturer who arouses the interest and curiosity of the students even though his demonstration apparatus may be built by his own hands and the room shared with chemistry, biology or history and American literature. Without him the best lecture room and the most elaborate equipment may be ineffective.

Laboratory Electric Services for the Undergraduate Physics Laboratory

CARL E. HOWE
Oberlin College, Oberlin, Ohio

IN this discussion the phrase *laboratory electric services* is interpreted as referring to electric services furnished to instructional laboratories, classrooms, research rooms and work rooms for the purposes of instruction, demonstration and research. It includes sources of various types of electric service as well as distribution systems and panels. No reference is made to building light and power circuits except as they directly concern the problem at hand.

Since the electric service problem is different for each individual building, no attempt is made to present a plan that would necessarily be satisfactory for all laboratories. Rather, there are described in some detail the electric services which are installed in the Oberlin College physics building.[1] This installation was made in the light of experience gained with a similar experimental system that had been in use for a period of ten years in an old building. It is hoped that the plan described here contains information of value to physicists in planning electric installations for the undergraduate physics laboratory.

The present installation really consists of three separate systems: (1) a large main distribution system, for distributing various direct and alternating power voltages throughout the building; (2) a moderately sized "communication" network for the distribution of voice frequencies and low-level signals; and (3) a limited amount of coaxial transmission line for the distribution of radio frequencies.

[1] Occupied February 1943.

MAIN DISTRIBUTION SYSTEM

The heart of this system is the main control and cross-connecting switchboard located in the generator and switchboard room on the ground floor. Figure 1 shows the plan of this board, which is 15 ft, 3 in. long and 7 ft high. Two of the panels are used for battery charging, two for motor generator control and three for making the connections for the distribution of the electrical services to other parts of the building.

Sources of Power

Edison battery: 115 v, 75 amp hr capacity.

Lead batteries: 120 v, 100 amp hr capacity; 30 v, 115 amp hr capacity; 12 v, 115 amp hr capacity; 6 v, 300 amp hr capacity.

Generators, direct current: 150 v, 10 kw; 40 v, 1.2 kw; 7½/15 v, 900 w.

Generator, 500 cycle/sec: 100 to 300 v, 500 w.

From power lines: 115/230 v, 60 cycle/sec, single-phase a.c.; 230 v, 60 cycle/sec, three-phase a.c.

Timing contacts: minutes, seconds and interrupted seconds.

The storage batteries are housed in a room adjacent to the generator and switchboard room.

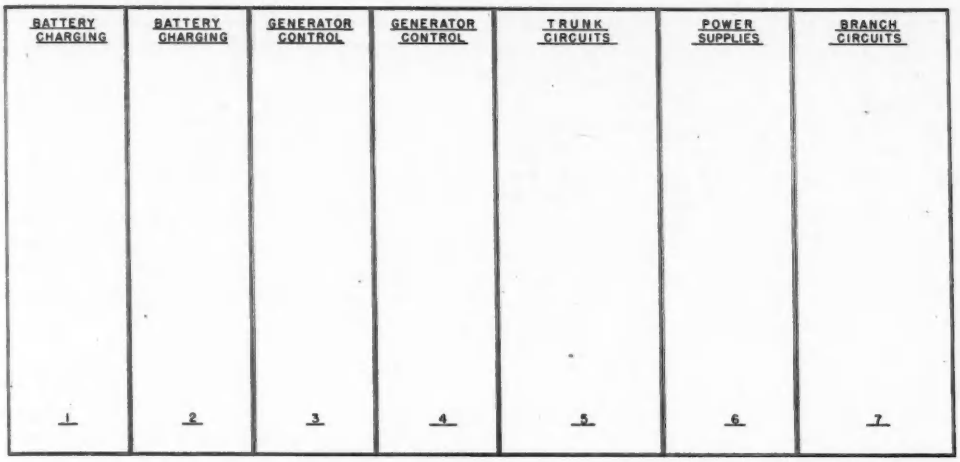


FIG. 1. Plan of the control panels and main distribution board.

Except for the 6-v battery, the lead batteries consist of banks of ordinary 6-v automobile batteries. Experience has shown that, with regular attention and when used as units (no intermediate taps), the batteries will have a useful life of at least six years. Arrangements are made for charging the batteries either with electronic rectifiers or with motor generators. The negative terminals of all batteries are connected to ground.

The motor generators are mounted in the same room with the switchboard. They are started and stopped by means of push buttons on the control panels. A time clock automatically turns off the chargers and the motor-generator sets at 5:55 P.M. and at midnight. Reverse-current relays prevent the batteries from discharging through the generators when they stop.

A synchronous timer operates relays which close contacts for approximately $\frac{1}{5}$ sec every second and every minute. In addition, there are provided seconds "pulses" which are interrupted at the half-minute and at the minute in a fashion similar to Government time signals. These timing pulses are very convenient, particularly where readings are to be taken every minute. They may be used to flash a light, to control an audio signal or to operate a magnetic counter.

Main Distribution Panels

On the Power Supplies panel, Fig. 2(*b*), appear multiple-connected female jacks for each of the sources available. The timing terminals have one male jack and one female jack so that the timing relay contacts may be connected in series with any desired source. At the bottom of the panel are two sets of multiple-connected jacks, a pair of male jacks at the top. These multiple connections may be used for obtaining additional outlets for any desired source or for making interconnections between outgoing circuits located on the panels on either side of the Power Supplies panel. There are also located at the lower left-hand corner of this panel connections to a magnetic switch which may be connected into any desired circuit for remote control switching of a heavy, inductive load.

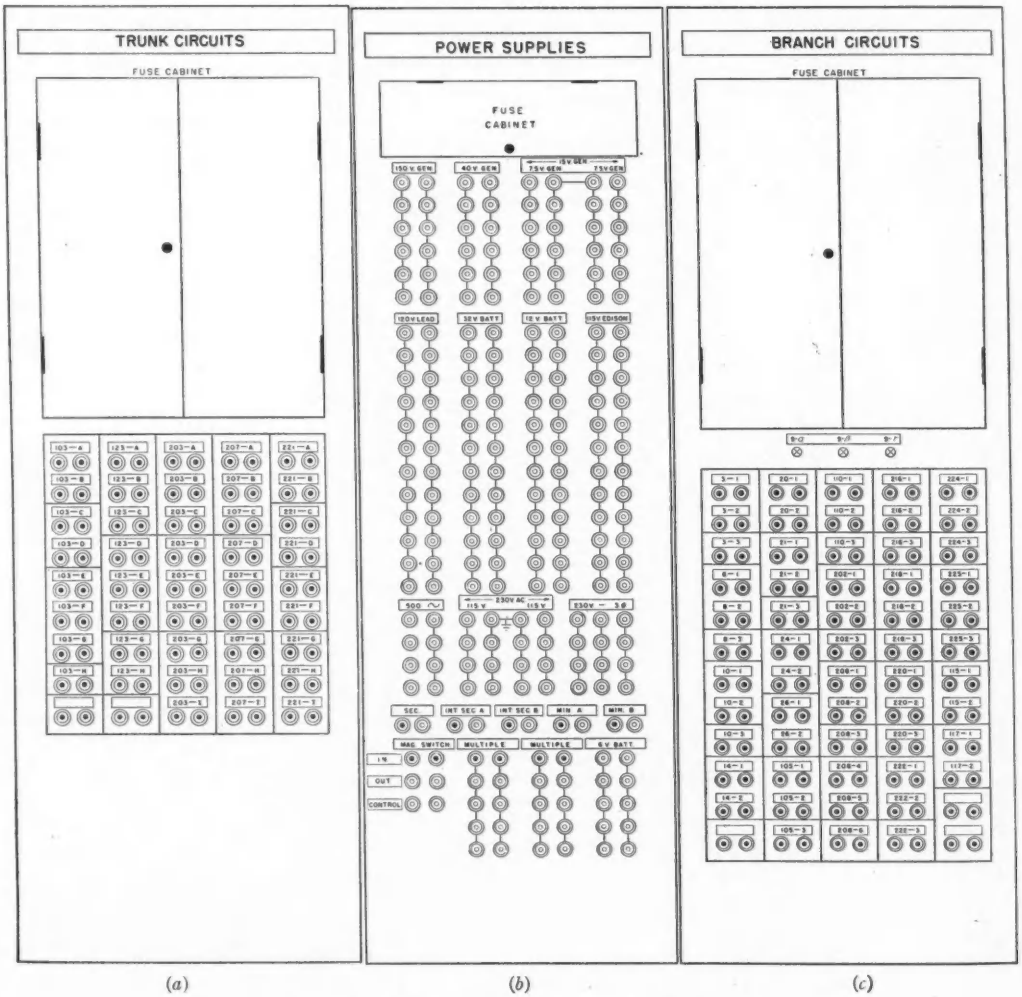
Outgoing circuits are located on panels on either side of the Power Supplies panel. On the Trunk Circuits section, Fig. 2(*a*), are pairs of male jacks connected to the trunk circuits going to rooms 103 (lecture room), 123 (first-year laboratory), 203 (light and atomic physics laboratory), 207 (heat and intermediate physics laboratory) and 221 (electrical measurements laboratory). In each case the circuits going to any one room are arranged in a vertical column. The trunk lines are easily identified by their labels; for example, *103-A* designates trunk line *A* going to room 103, and *203-G* is trunk line *G* of room 203. A rectangle outlined about a group of jacks indicates that those particular wires are carried in a separate conduit to their destination. Thus the wires for lines *103-A* and *103-B* are in one conduit, *103-C* and *103-D* in another, and so on. With this arrangement it is possible to set up circuits in such a way as to minimize "pick up" from one circuit to another. Fuses for these lines are contained in the fuse cabinet at the top of

the panel. These fuses are of the 30-amp screw-in plug type with the exception of the 60-amp cartridge fuses for circuit *103-H* going to the lecture room.

Outgoing circuits to smaller rooms are located on the Branch Circuits panel, Fig. 2(*c*). In most instances there are three circuits to each room, the label above each pair of male jacks indicating first the number of the room and second the number of the particular circuit connected to that pair of jacks. Thus room 224 has three circuits going to it; *224-1*, *224-2* and *224-3*. Again a rectangular outline about a group of jacks indicates that the wires connected to those jacks are carried to their destination in a separate conduit.

At the center of this panel are three communication jacks, labeled *9-α*, *9-β* and *9-γ*. These jacks are connected to shielded wires which form a part of the "communication" network to be described later.

Throughout the entire electric distribution system it is the invariable rule that the left-hand side of a pair of jacks or terminals shall be negative and the right-hand side positive for direct currents. Consequently, for outgoing cir-



(*a*) (*b*) (*c*)

FIG. 2. Panels for power supplies and outgoing circuits.

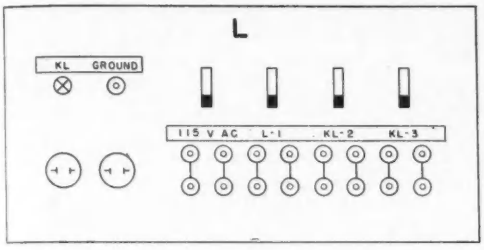


FIG. 3. Wall-mounted student panel in the electrical measurements laboratory.

cuits the left-hand jack is colored black and the right-hand one red. This color scheme is deviated from to a certain extent on the Power Supplies panel. In general these deviations are: light gray for the grounded negative battery connections; white for the grounded a.c. connections; green for the "hot" a.c. connections; orange for three-phase connections. Whenever possible the grounded a.c. connection is kept to the left.

The male and female jacks used on these panels are of the Standard Electric Time Company DF-1 type with 75-amp capacity. Connecting cords of flexible rubber-covered copper wire, 70 each of No. 10 and No. 8, are furnished with a male plug on one end and a female plug on the other. Half of these cords have red and half have black rubber handles. They are provided in lengths of 4, 5 and 6 ft. Wall-mounted cord racks permit the storing of cords not in use. By means of these cords any service can be connected to any outgoing circuit. It is impossible to short circuit any source, and the board is completely "dead front."

The setting up of circuits on these panels is limited to staff members and one or two trained assistants. To avoid the accumulation of a large number of cross connections that are not being used, a supply of tags is furnished on which is written the "end date" of use and the name of the individual who set up the circuits. Each week the board is cleared of the excess connectors.

Student Panels

In each of the four large instructional laboratories is a subdistribution panel for connecting the electric services from the trunk lines to the lines going to the individual student panels. The use of the subdistribution panels can be more easily explained after the student panels are described.

Figure 3 shows one of the student panels of the electrical measurements laboratory. This particular panel is identified by the letter *L*, the remainder of the panels by other letters of the alphabet. There are four toggle switches on this panel. A single-pole switch turns on the 115-v a.c. outlets (jacks) beneath it. There are three circuits coming from the subdistribution panel, each controlled by a double-pole switch. The first circuit is labeled *L-1*, indicating that panel *L* is the only one on which this circuit terminates. The second circuit is labeled *KL-2*, indicating that this circuit terminates not only on panel *L* but also on panel *K* (located at another position in the laboratory). Similarly, circuit *KL-3* is common to panels *K* and *L*. There are also two a.c. convenience outlets which are not controlled by the a.c. switch, a *Ground* terminal and a communication jack, the circuit of which is common to panels *K* and *L*. This type of panel is mounted flush with the wall. Figure 4 is a photograph of a wall-mounted panel in the first-year laboratory.

Figure 5 is a photograph of a panel mounted on one of the tables in the middle of the electrical measurements laboratory. The metal box on which it is mounted has another panel on the opposite side so that students may work on both sides of the table. The lower terminals are 6 in. from the top of the table, making them accessible even when resistance boxes or similar pieces of equipment are placed right in front of the panel. On the end of the box may be seen one of the

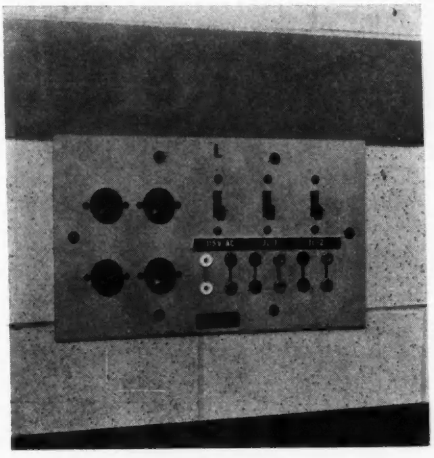


FIG. 4. Wall-mounted student panel in the first-year laboratory.

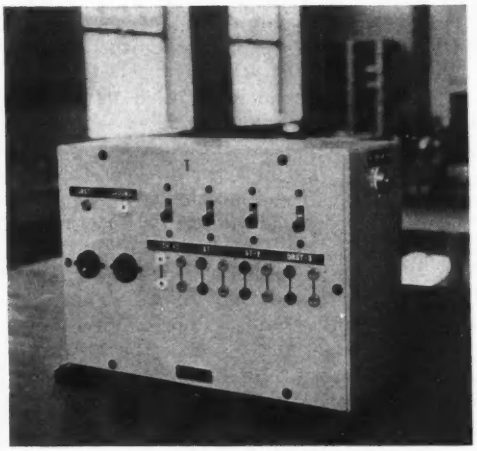


FIG. 5. Table mounted student panel in the electrical measurements laboratory.

three coaxial cable connectors terminating in this laboratory.

It may seem odd that there are no binding posts on these panels, or any others throughout the building. The reason is that we have gone completely to a plug-and-jack system of making electric connections. Our staff is unanimous in acclaiming the success of this venture, the details of which are too involved to describe completely here.

The female jacks used on the student panels are Standard Electric Time Company type No. DF-10 of 10-amp capacity, which have been reamed out to the same size as the General Radio jacks described below. For the laboratory equipment, General Radio Type 274-J jacks are used. This equipment includes all electric apparatus such as resistance boxes, rheostats, potentiometers, capacitors, inductors, sockets, meters, oscillators and power supplies. Connecting cords are made of flexible insulated wire and General Radio Type 274-P plugs with a special Bakelite handle. The task of completing this project has not been an easy one. Some idea of its size may be gained from the numbers of jacks and plugs involved—approximately 6000 jacks and 2500 plugs.

There are 23 student panels in the electrical measurements laboratory and three small adjoining rooms. The arrangement of these panels is shown in Fig. 6. This arrangement is typical of all of the other instructional laboratories, except that in the latter there is but one double-faced panel on each table in the center of the room. There are 23 student panels associated with the first-year laboratory, 16 in the heat and intermediate physics laboratory and 18 in the light and atomic physics laboratory.

The student panels in the other advanced laboratories are identical with those in the electrical measurements laboratory except there

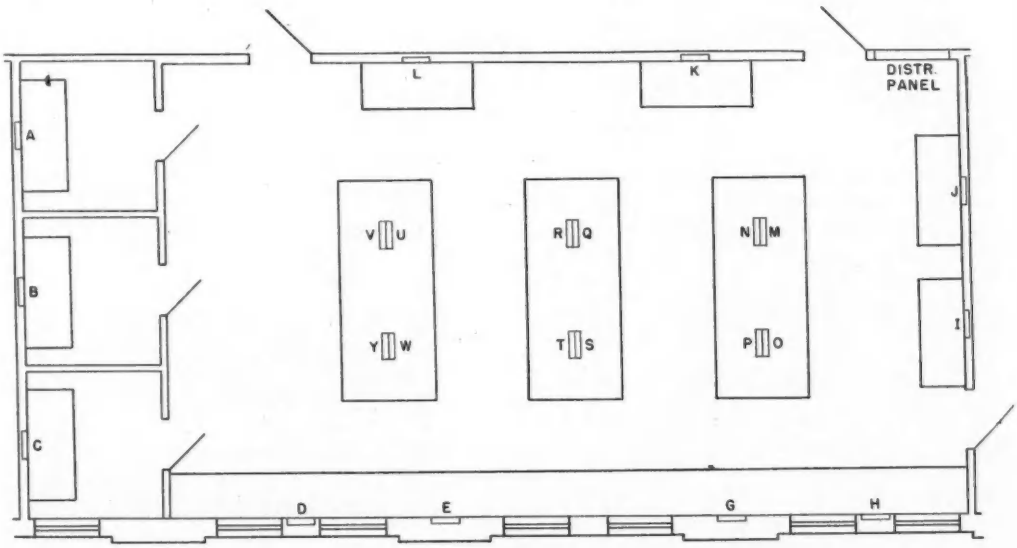


FIG. 6. Arrangement of student panels in the electrical measurements laboratory.

are no connections to the communication network. The panels of the first-year laboratory are smaller, having only two circuits going to the subdistribution panel, no communication jack and no *Ground* connection. There are, however, four convenience outlets to each panel. Figure 4 shows one of the panels mounted flush with the wall in the first-year laboratory.

Subdistribution Panels

Each of the four large instructional laboratories has a subdistribution panel somewhat similar to that shown in Fig. 7. The one represented here serves room 221, the electrical measurements laboratory. The left-hand side of the panel takes care of the incoming electric services and switches, while the right-hand side has fuses and jacks for 43 circuits going out to 23 student panels.

The topmost row of switches are single-pole 25-amp circuit breakers for turning on 115 v a.c. for the student panels. Each breaker controls the circuit going to the panels indicated by the letters on the label above it. The next two rows of switches (double-pole) are for the nine trunk lines and the 115-v d.c. circuit coming from the main distribution panel. The three switches on the bottom row control the single-phase and three-phase circuits to the corresponding jacks on the panel.

All single-phase power lines to subdistribution panels come through magnetic switches mounted on the main light and power panel on the ground floor. The two momentary-contact push buttons below the pilot light underneath the *AC 1φ* label are used to turn the corresponding magnetic switch on or off. In case the circuit has not been turned off already, the magnetic switch is released automatically when the time clock turns off the battery chargers and generators. This insures that no soldering irons, electric heaters, power supplies, and so forth, will be left on overnight. No such arrangement has been made for other services, but it could be done if desired.

To the right of the push-button control are three sets of multiple-connected communication jacks which go to the central communications network panel.

Below the switches are the multiple-connected female jacks connected to the trunk lines and 115-v d.c. circuit coming from the main distribution panel. Again a rectangle outlined about a group of jacks indicates that the wires feeding

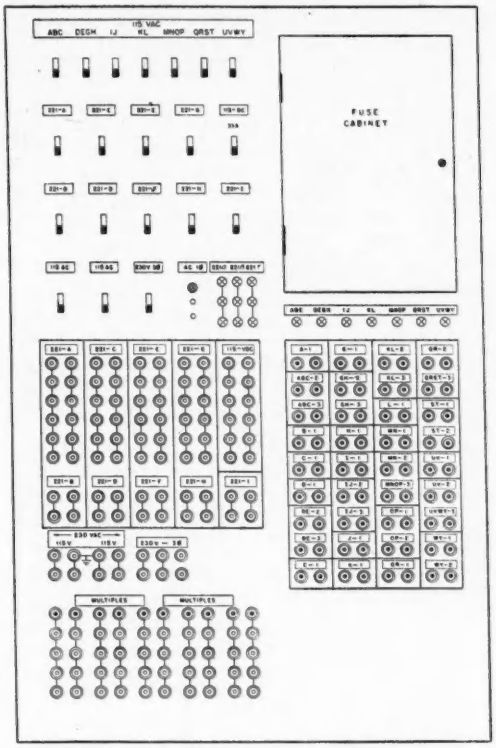


FIG. 7. Subdistribution panel in the electrical measurements laboratory.

them come through a common conduit from the main panel.

At the bottom of the panel are five sets of multiple connections which serve the same purpose as the corresponding ones on the Power Supplies section of the main distribution panel.

The right-hand side of the subdistribution panel is used for the circuits going out to the student panels. The fuses for these circuits are in the cabinet at the top of the panel. Below the fuse cabinet are seven communication jacks, each of which is connected to corresponding jacks on the panels indicated by the letters on the label. By means of connecting cords with plugs, these circuits may be connected to each other or to the communications network by using circuits *221-α*, *β* and *γ*.

Below these communication jacks are the male jacks for the remainder of the circuits going out to the student panels. In each case the labeling is such as to indicate exactly where the circuit terminates. Thus, *A-1* indicates a circuit termi-

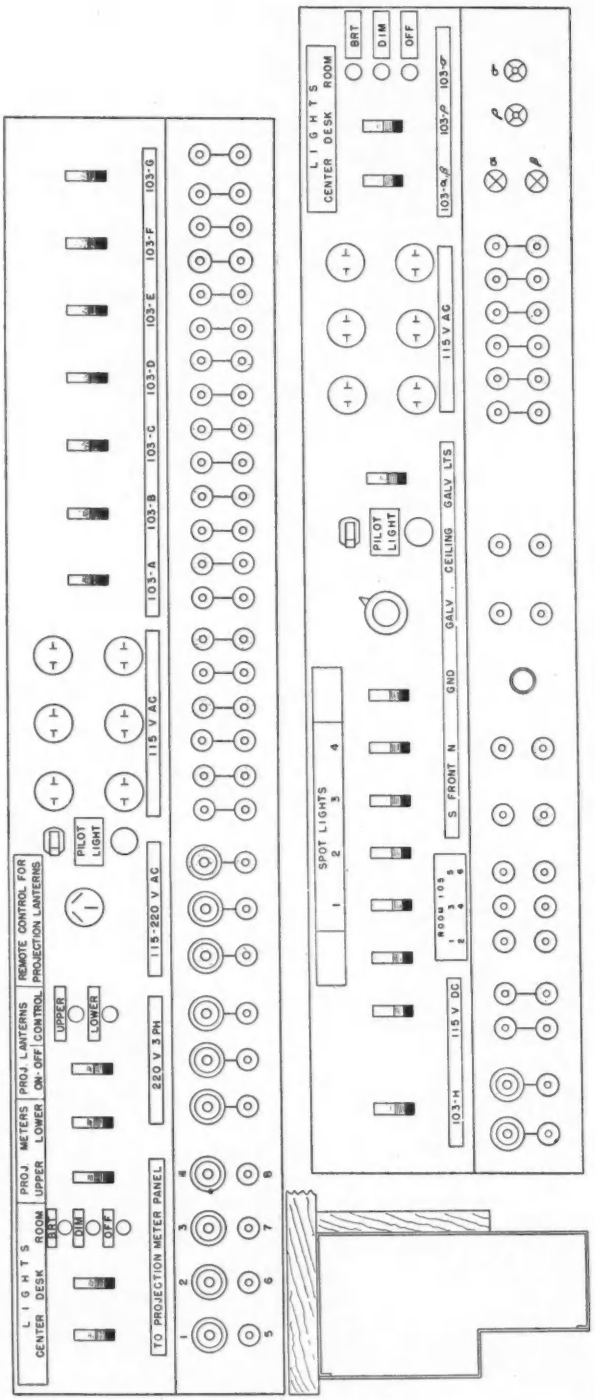


FIG. 8. Electric services panel for the lecture room.

nating at position *1* on panel *A*; *ABC-2*, a circuit terminating at position *2* on panels *A*, *B* and *C*; and *QRST-3*, a circuit terminating at position *3* on panels *Q*, *R*, *S* and *T*. This method of labeling gives an added precautionary measure. Suppose it is desired to get 115 v d.c. on circuit number *3* of panel *L*. This circuit is labeled *KL-3* (see Fig. 3). Before making the connections at the subdistribution panel, circuit *3* on both panels *K* and *L* should be checked to see if any equipment has been left connected to this circuit. After this check is made, connecting cords, with a male plug on one end and a female plug on the other, are used to connect the *KL-3* male jacks on the subdistribution panel to one pair of the female jacks of the 115-v d.c. circuit.

The importance of this procedure can be illustrated by the following example. A student at panel *Q* is having trouble with "pick up" from a filament transformer and decides to place the transformer at a greater distance from the rest of his equipment. A simple way is to take the transformer to another panel, say *S*, connect the primary to the 115-v a.c. terminals at that panel and connect the secondary to circuit *QRST-3*. Then at his own panel, *Q*, filament connections are made to this same circuit. This procedure is quite legitimate if the circuit is not needed for something else. Now suppose a student working at panel *T* requests that he be given 115-v d.c. on circuit *QRST-3*. The connections already made to this circuit do not appear at the subdistribution panel. If the 115-v d.c. were connected to this circuit without checking, equipment would be burned out.

On the subdistribution panels Standard Electric Time Company, type DF-5 jacks of 40-amp capacity are used. The connecting cords are similar to those used with the main

distribution panels but have DF-5 plugs and No. 10 wire.

The backs of the subdistribution panels are flush with the opposite side of the wall in which they are mounted. Removable steel cover plates make the wiring readily accessible.

Lecture Room Services

The main electric services panel for the lecture room is approximately 7½ ft long and 10 in. high. It is mounted on the instructor's side of the 12-ft lecture desk just underneath the table top. Figure 8 shows this panel as two separate sections, displaced for the purpose of reproduction. These two sections are actually mounted end to end. There is also shown a cross-sectional view of the panel and its box in position. The lower part of the panel is recessed to avoid having the plugs with connecting cords interfere with the lecturer. All of the jacks are mounted on this recessed portion.

At the extreme left of the panel are two switches for turning on a single light in the center of the room and the lights over the desk and blackboard. The main lights for the lecture room are fed through a motor-driven Autrastat dimmer which is controlled by the three momentary-contact push buttons under the label *Room*. Three positions are available: bright, dim, and off. The dim position gives just enough illumination for students to take notes while lantern slides are being shown. Since there are no windows in the lecture room it can be made completely dark by merely turning off the lights. If the room is in complete darkness when the *Bright* button is pushed, the lights immediately come on dim and then slowly increase in intensity until full illumination is attained. This procedure permits the eyes to adjust themselves without discomfort to what would be otherwise a harsh change of illumination. If now the *Off* button is pushed, the lights go to dim illumination and then off. If the *Dim* button had been pushed, they would have gone to dim illumination and stayed there. All of these light switches and controls are duplicated on the extreme right-hand side of the panel and at a control panel at the rear of the room.

Next on the panel come two switches for turning on and off projection lanterns each of which is equipped with a projection meter. These two projection units are mounted one on top of the other at the rear of the room. Underneath the light controls are eight jacks connected to similar jacks on the control panel near the projection units. These circuits permit electric connections between the desk and the projection meters.

The next switch controls the power to two projection lanterns for showing slides. These lanterns are mounted one on top of the other at the rear of the room. Only one lantern can be on at a time, the change being made either by pushing the proper control button or by means of a push button on an extension cord plugged into the remote-control plug provided. The change from one lantern to the other gives a dissolving effect which is not unpleasant. The push-button switch controls for the projection equipment are duplicated on the panel at the rear of the room.

Pilot lights with accompanying switches permit faint illumination of the panel when the room is darkened. The alternating-current outlets need no explanation. The eight trunk lines, *103-A*, *103-B*, etc., and the *115 V DC* circuit

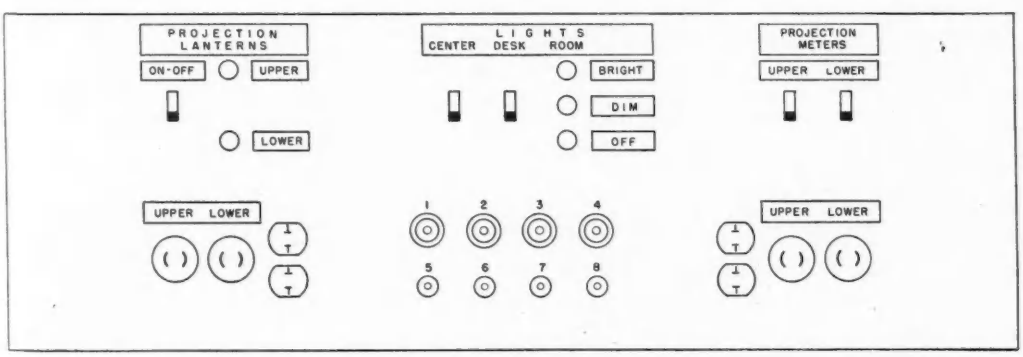


FIG. 9. Control panel at the rear of lecture room.

from the main distribution panel are provided with double-pole switches. Circuit *103-H* is designed to carry larger currents than the others and is provided with a remotely controlled magnetic switch which is mounted on the main distribution panel on the ground floor.

Four switches are provided for turning on and off groups of spotlights mounted on the ceiling to provide additional spot illumination of equipment on the lecture desk. A spare switch is provided on either side of this group of switches.

Underneath these switches are six jacks which are connected to wires going to an adjacent room No. 105. There are also two pairs of jacks leading to small panels on the front wall of the lecture room, one panel on either side of the blackboard. These same panels have a.c. convenience outlets, the circuits to be used for the operation of any electric equipment mounted on the front wall for demonstration purposes.

On the ceiling above the lecture desk is a small panel with three pairs of outlet jacks. One pair is used to carry current to a reflecting galvanometer mounted near the panel on the ceiling; a spot of light reflected from this galvanometer falls on a scale at the top of the lantern screen at the front of the room. The second pair of jacks furnishes the power for the galvanometer lamp. The third pair of jacks is used in connection with any electrically operated or controlled equipment mounted on the ceiling, for example, a Kelvin wave model. The switch for turning the galvanometer lamp on, jacks for the other two circuits and a potentiometer for controlling the galvanometer sensitivity are mounted at the indicated places on the lecture-table panel.

Underneath the light switches at the extreme right-hand end of the panel are two communication jacks, *103-α* and *103-β*, and two coaxial cable jacks, *103-ρ* and *103-σ*. These circuits lead to the corresponding distribution panels located elsewhere in the building.

Near the rear of the room is mounted a panel (Fig. 9) with the duplicated light and projection-equipment controls and with the power outlets for the projection lanterns. The lantern-slide projectors are on a table to the left of the control panel, the two projection meter units being on the right.

Fuses for the circuits of the lecture-room panel are located in a separate cabinet mounted at a convenient place on the wall in an adjoining room. Fuses may be changed by an assistant without distracting the students.

Classrooms

A small panel with services similar to those of the first-year laboratory student panels (Fig. 4) is mounted on the end of the lecturer's desk in each of two classrooms. The circuits come directly from the main distribution panel. Fuses are contained in a cabinet with a hinged cover at the back of the panel.

Display Cabinets

On opposite sides of the main entrance are two large display cabinets for exhibits which are changed weekly. Two circuits to each cabinet from the main distribution panel permit the operation of electric equipment.

Research Rooms and Work Rooms

In Fig. 10 are shown the electric services provided for the research rooms and work rooms. The large center panel has the same services as

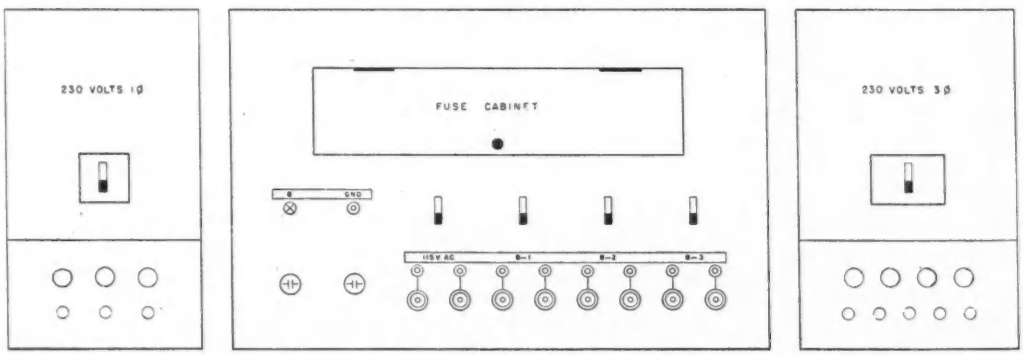


FIG. 10. Electric services provided in research rooms and work rooms.

those provided on the student panels in the electrical measurements laboratory. The larger jacks, DF-5, are furnished to take care of large currents, the small ones being retained to permit the use of regular laboratory equipment. The circuits come from the main distribution panel. Fuses are contained in the cabinet as indicated. There are 14 of these panels located in various rooms.

To the right of this panel is a three-pole circuit breaker for three-phase power. Knockouts are provided in the removable lower part of the cover of the box. These may be used for jacks, binding posts or conduit connections. On the left side of the panel is a two-pole circuit breaker for 230-v single-phase power.

Just below the large panel, and not shown in Fig. 10, is an outlet box with cover containing knockouts. This box is connected by a conduit to a similar box on the other side of the room. There is room for six conductors to take electric services to the other side of the room.

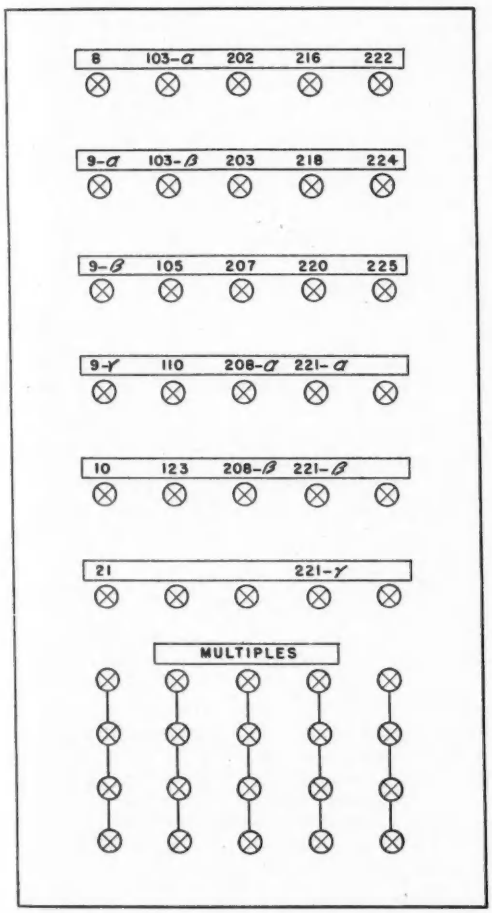


FIG. 11. Panel for communication network.

COMMUNICATION NETWORK

The communication network circuits are all brought to the panel shown in Fig. 11. Each circuit consists of three No. 18 copper wires twisted together into a cable, passing through a separate conduit and shielded between conduit and panel with a grounded flexible metal tube. One of the wires is connected to ground. Each circuit is terminated at either end with a Mallory-Yaxley three-circuit microphone jack, long-frame type, Cat. No. 2B. Cross connections are made with patch cords consisting of flexible rubber-covered two-conductor cable with a braided copper shield and Mallory-Yaxley three-way microphone plugs with shielded nickel shell, Cat. No. 76A. This arrangement gives a three-conductor circuit with two conductors balanced to ground.

COAXIAL CABLE

A limited amount of coaxial cable (approximately 1000 ft) has been installed for the transmission of high-frequency alternating currents up to approximately 100 megacycle/sec. All but one of the cables are terminated on the panel shown in Fig. 12. Two of them go to the lecture room (103), two to a small special laboratory (208), two to the roof for antennas and three to student panels on center tables in the electrical measurements laboratory (221). Room 208 has an additional line to the roof.

The cable used is Amphenol copper tubing transmission line No. 72-12C. This cable consists of a copper tube of $\frac{3}{8}$-in. inner diameter with an inner conductor of No. 12 solid copper wire. The insulation is Amphenol No. 73 polystyrene beads. The characteristic impedance of the line is 72 ohms. These transmission lines are terminated with chassis-type female connectors No. 93-C. Cross connections are made with flexible coaxial cable No. 72-12 and cable-type male connectors No. 93-M.

Standard Frequencies

The panels shown in Figs. 11 and 12 are mounted with other equipment in a Standard Frequency Room. Standard frequencies are obtained from a 100-kilocycle/sec quartz crystal

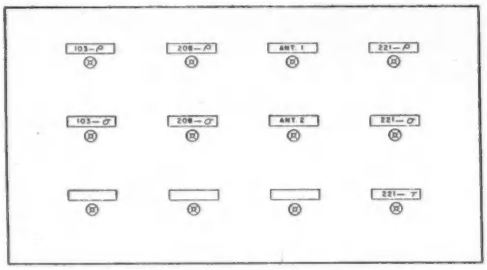


FIG. 12. Panel for coaxial transmission lines.

oscillator mounted in a double oven, the temperature of the crystal being maintained constant to within 0.01°C. The frequency of the oscillator is periodically checked against the 5-megacycle/sec carrier of WWV. Multivibrators have been made to operate at frequencies of 10^5, 10^4, 10^3, 10^2 and 25 cycle/sec. The frequencies are used for radiofrequency and audiofrequency calibrations and for accurate timing. They are used with equipment in the same room or sent to other rooms by means of coaxial cables or communication circuits. The 25-cycle/sec frequency makes a very satisfactory control for spark recording, such as is used in the Behr free-fall apparatus.

Ground Connections

The jacks labeled *Ground* on the service panels go to a ground which is separate from that of the negative terminals of the storage batteries. The ground for these jacks was made by burying two copper plates, 4×6 ft, underneath the building. The plates, placed vertically and parallel to each other and approximately 1 ft apart, were completely surrounded by very fine sand (molders sand). Separate connections from each plate are brought out to a terminal board which is arranged in such a way as to permit the measurement of the resistance between the plates, thus getting an indication of the resistance to ground. Through a pipe connection water can be introduced to the area about the plates in case it is ever necessary to decrease this resistance.

Size of Conductors

All trunk lines from the Trunk Circuits section of the main board are No. 8 copper wire. For the circuits going out from the Branch Circuits section, No. 8 wire is used for the No. 1 circuits, No. 10 wire for the No. 2 circuits and No. 12 wire for the No. 3 circuits. No. 12 wire is used for the circuits going from the subdistribution panels to the student panels.

Our department has found the foregoing arrangement very satisfactory. In the two years it has been in use only several minor additions have been desired. The 6-v battery and the remotely controlled magnetic switch on the Power Supplies section of the mainboard have been added since the original installation. We have not regretted the replacement of the 300-v battery (a part of the old system) by a number of portable, variable voltage, electronically regulated power supplies. They have been much more satisfactory for use in the electronics work and are far less expensive. In the old installation we had on the main distribution panel a source of stabilized 115-v a.c., obtained from a 500-w voltage stabilizer. This was discarded in favor of small portable stabilizers.

It has been the author's privilege to have been closely associated with this project. It has, however, been distinctly a departmental undertaking, not exclusively a personal one. Our ideas have been implemented extensively by the wide experience of the Standard Electric Time Company.

IF they ever think at all about their place and function in society, scientists may be inclined to overestimate their importance as shapers of public opinion and educators of the mass of mankind. The mass of mankind knows next to nothing of them or their work, and if it knew more, might think even less. What little it does know fosters a sullen distrust. Men who hold their hypotheses lightly or who, like Newton, glory that they frame none, are not popular. They never were. And while science goes its indifferent way, the world it would serve yearns for the futilities of a nostalgic humanism that knew better days three hundred years ago, and surrenders its intelligence to the unreason of credulous mysticisms.—E. T. BELL, *Am. Math. Mo.* **49**, 572 (1942).

NOTES AND DISCUSSION

Completely Inverted Images

PAUL KIRKPATRICK
Stanford University, Stanford University, California

EVERYONE knows that a simple telescope makes things appear upside down and transposes them from left to right and *vice versa*. But does it also reverse things fore and aft, that is, along the line of sight? After witnessing the following demonstration many observers will agree that it does.

Set up a simple telescope of the kind used for galvanometer observation and focus it on a rough plastered wall with projecting sand grains. To make the sand grains conspicuous, let the wall be illuminated obliquely, as by a window at one side. On looking through the telescope one sees not projecting particles but indentations!

Cider from the Newtonian Apple

RICHARD M. SUTTON
Haverford College, Haverford, Pennsylvania

IT seems obvious that the gravitational pull of the earth on the moon must exceed the pull of the sun on the moon because the moon is so near and the sun so far; and, besides, the moon is *our* special satellite, revolving around the earth. But not all "obvious facts" are true, and it gives one a start to discover that such a comfortable but unconsidered belief is false.

In these days of super-rockets, it seemed to me timely to compute the distance from the earth at which the gravitational pull of the sun on a particle of matter is equal to the gravitational pull of the earth on the same particle. One never knows when such information may be useful! I was surprised to find, as all astronomers doubtless know,[1] that the distance is about 161,000 mi, or only about two-thirds as great as the average distance from the earth to the moon. Our moon, therefore, is *constantly* under a greater pull from the sun than it is from the earth; in fact, the pull of the sun is more than twice as large as that of the earth.[2]

Some readers may find amusement for themselves and their students in considering the query: Why does not the sun therefore steal the moon from the earth? Has the moon worn such a deep groove in space that it does not respond to this larger force?

If this question has not occurred to you before, don't answer it too glibly. Consider, for example, the situation at new moon when the moon is between the sun and the earth. The resultant force on the moon is definitely *toward the sun*. Now, then, can the moon describe a path which, in two weeks time, brings it to the side of the earth away from the sun at full moon?

[1] See, for example, F. R. Moulton, *An introduction to celestial mechanics*, p. 362, prob. 1.

[2] At 93,000,000 mi from the sun, the gravitational field strength due to the sun is approximately 0.60 dyne/g; at 24,000 mi from the earth, the field strength due to the earth is only 0.27 dyne/g.

Unusual Rainbows

FREDERIC PALMER
Haverford College, Haverford, Pennsylvania

TO what extent are all rainbows alike? Common experience tells us that they appear to differ in length of arc, in brilliance, in altitude, and in the presence or absence of a secondary bow and supernumerary bows; but all these variations may be accounted for by differences in the characteristics of the local shower and the altitude of the sun, and by applying the elementary theory of bow formation which we teach our students. If we wish to inquire further into the matter, we may turn, for example, to Preston's *Theory of Light* from which, unfortunately, no additional information is to be obtained—a fact which may tend to make us believe that no other types of bow are possible. Yet R. W. Wood, in his *Physical Optics*, mentions that a rainbow with a prominent yellow band through the middle is not an uncommon phenomenon.

Possible variations in the appearance of the rainbow are summarized by W. J. Humphreys[1] as follows:

> The records of close observations of rainbows soon show that not even the colors are always the same; neither is the band of any color of constant angular width; nor the total breadth of the several colors at all uniform; similarly the purity and brightness of the different colors are subject to large variations. The greatest contrast, perhaps, is between the sharply-defined brilliant rainbow of the retreating thunderstorm and that ill-defined faintly tinged bow that sometimes appears in a mist—the "white bow" or "fog-bow".

Tyndall[2] describes several such fog-bows and says that attention was first called to the phenomenon by Don Antonio de Ulloa. Tyndall records his own observation of such a bow when he stood with a bright lamp behind him looking out of an open door into the fog. The bow then had the appearance of a white halo about the shadow of his head.

The conditions under which a fog-bow can be formed with the sun as the light source are so critical that the phenomenon must be one which is only rarely observed. These conditions are: (1) the sun must be within approximately 30° of the horizon; (2) the fog droplets must be less than about 6×10^{-3} cm in diameter; (3) either the density of the fog must be small or the observer must be near the edge of the fog bank so that the intensity of the sun's rays in their original direction are not too greatly reduced by scattering.

The object of this note is to record the observation of a beautifully formed fog-bow in sunlight during August at Wellfleet, Cape Cod, Massachusetts. I arose at 7:30 A.M. and went for a prebreakfast dip in the waters of Wellfleet Bay, which were placid at the time and covered with a blanket of thin mist some 20 ft in thickness. The sun, still tinged with its sunrise color, was about 10° above a cloudless

horizon. I waded in until the water was above my waist and then happened to glance toward the west. The appearance of the fog-bow which I then saw was striking. It was brilliant white against a background of gray mist and looked like a "child's size" bow made of absorbent cotton with ends dipping into the water. The overlapping of the colors was so complete that the bow appeared to be white up to the edges. The angle between the horizontal and a line from the eye to any point on the bow was not far from 30°. Hence, if the sun's altitude was 10°, this line made with a line from the sun through the eye an angle of about 40°, which is approximately the angle at which such a bow could be formed under these conditions. In comparison with the usual primary rainbow, the fog-bow was not only lacking in color and of much smaller radius, but it appeared also to be much nearer, perhaps 20 yd away.

In his *Meteorology* (p. 338) A. E. M. Geddes describes the fog-bow as follows:

> The white band in the rainbow arises from the overlapping of colors. This occurs when the drops are very small, and for drops with a radius $=30\times10^{-4}$ cm or less the region between 40° 40′ and 38° 40′ from the anti-solar point appears white. As this bow is due to minute drops, and occurs during fog or mist and not during rain, it might almost better be styled a mist-bow than a rainbow.

I wish to record seeing, in July from the same location at Wellfleet, a bow of such a nature that it must be much more rarely observed even than the fog-bow, namely, a red rainbow. Indeed, I have failed to find in our standard works any mention of the possible existence of such a bow. The circumstances under which this bow was observed are as follows. There was a sharp shower for about 15 min just as the sun was sinking toward a cloudless western horizon. Looking toward the east from a point about 80 ft above Wellfleet Bay, I saw an unusually brilliant primary bow at its maximum altitude, with one end in the water and the other end on the shore. As I watched, the blue, green, yellow and orange portions were quickly wiped out, the entire operation taking place in not more than 1 sec. There remained a bow of a single color, red, only slightly less brilliant than before, and in width about a quarter of that of the original bow. I turned to the west and found that the sun had disappeared completely below the horizon, though the familiar red afterglow was still strong. No further change took place in the appearance of the rainbow for, perhaps, 30 sec, when it suddenly vanished. A bow of but a single color was visible when there was no sun in the sky!

The morning fog-bow I had estimated to be at a distance of about 20 yd from the eye. Since the droplets which participated in the formation of the bow may be at various distances from the observer, it is not easy to say what determines this estimate of distance. Tyndall[2] says "The apparent distance of the circular bow is often great in comparison with that of the originating drops. Both distance and diameter may be made to undergo variations. In the rainbow we do not see a localized object, but receive a luminous impression, which is often transferred to a portion of the field of view far removed from the bow's origin." In the case of the Wellfleet rainbow, this distance was easily estimated since one end of the bow rested upon the shore. Certain houses and dunes appeared to be seen through the bow; that is, they appeared as if covered by a colored veil, whereas others, nearer the observer, appeared just as definitely uncolored. The line of demarcation between the two was not difficult to locate, and since distances along shore were well known, it was easy to estimate that the distance from the observer to the line of demarcation—to the line where the bow began—was about half a mile.

[1] W. J. Humphreys, *Physics of the air* (ed. 1), pp. 456–457.
[2] J. Tyndall, *New fragments*, chapter on "The rainbow and its congeners."

Location of Virtual Images

JOSEPH P. HARPER
University of Scranton, Scranton, Pennsylvania

IN experiments on mirrors and lenses there is usually one part pertaining to the location of virtual images. The method of parallax which is suggested in most elementary laboratory manuals is difficult and confusing for many students because it requires a certain type of ocular coordination and concentration not involved in ordinary vision. In addition, visual defects frequently deter a student from obtaining accurate and consistent results. To overcome these difficulties the writer has for some time substituted the method outlined below for locating virtual images.

With an object O_1 (Fig. 1) between a convex lens L_1 and its focal point there will be a virtual image of O_1 at *VI*. An auxiliary convex lens L_2 introduced between lens L_1 and the screen S can be so placed that a real image of the virtual image appears on the screen. With lens L_1 removed, but with lens L_2 and the screen S in their original positions, the object can then be moved to a new position O_2 such that lens L_2 forms an image of it on the screen. Since the auxiliary lens and the screen have not been moved, the new position of the object is obviously the same as that of the virtual image formed by the lens L_1 when it was in position.

Assuming that the optical bench scale increases toward the left, and letting the symbols for the several elements represent their bench-scale positions, we see that the object distance p for lens L_1 is L_1-O_1 and that the virtual image distance q for this lens is O_2-L_1. From these values the focal length f of the lens L_1 may be computed by using the lens equation in the form $(1/p)+(1/q)=1/f$. Focal lengths so determined check more consistently—at least for

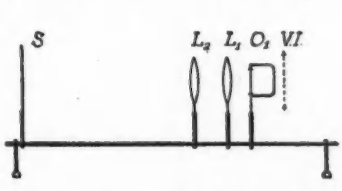


FIG. 1. Optical bench arrangement for obtaining a real image of the virtual image produced by lens L_1.

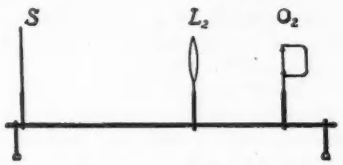


FIG. 2. Optical bench arrangement for locating the position of the virtual image; L_1 is removed and S and L_2 are in the same positions as before.

the elementary student—with those determined with real images than do those determined by the method of parallax.

A proper choice of lenses makes it possible to carry out this experiment on an optical bench. For accuracy in locating the position of a virtual image it is necessary to have the distance from the auxiliary lens to the virtual image about half the distance from this lens to the screen.

If desired, the ratio of the object size to the image size (for lens L_1) may be determined. The size of the object may be measured directly, and the size of the virtual image may be computed from the size of the real image (of the virtual image) on the screen and from the distances of the real and virtual images from the auxiliary lens. However, a more direct method is to obtain the ratio of the size of the second image (that of O_2) to the size of the first image (that of the virtual image) as they appear on the screen (Fig. 2).

A typical set of data obtained by an elementary student is as follows: screen at 100 cm, auxiliary lens L_2 at 36.4 cm, lens L_1 at 27.5 cm, original position of object O_1 at 20 cm, second position of object O_2 at 7.3 cm (when L_1 is removed); from these values there is obtained $p=7.5$ cm, $q=-20.2$ cm; the resulting focal length f is 11.9 cm; the corresponding value of the focal length as obtained by the same student when using real images is 11.8 cm.

The method is readily adapted to the case of concave lenses; however, it should not be considered as a substitute for the virtual object method employed in connection with this type of lens. With proper arrangements the method may also be applied to both concave and convex mirrors.

Although there is nothing new in this method it is apparently not used to any extent; yet it seems to have sufficient advantage over the method of parallax to recommend its use. It is also of value in demonstrating the fact that a virtual image may serve as an object for a lens or mirror as readily as a real object. The method can logically be presented after the student has learned the conditions under which a convex lens produces real images and after he has actually looked through the lens to see the virtual image on the object side of the lens.

DIGEST OF PERIODICAL LITERATURE

Demonstration of Molecular Motion

The apparatus herein described is designed to illustrate a wide range of properties of molecules as exemplified in the general concepts of the kinetic theory, equilibrium states, and the gas, the liquid and the solid phases.

A piece of $\frac{1}{4}$-in. plate glass, 4 in. square, is mounted in a heavy brass frame and surrounded with a molding of angle strips to serve as walls. This frame is suspended from a steel framework by four brass rods 6 in. long and $\frac{1}{16}$ in. in diameter, so as to form a horizontal table. The table has two natural modes of vibration—as a conical pendulum and as a torsional pendulum. It is made to vibrate with the frequency of either of these modes by a clockwork rigidly fixed to the table and carrying a heavy lead eccentric mounted on the last rotating arbor. The eccentric will turn at whatever rate the table is oscillated, and, once synchronized, the two will run until the spring is unwound. A leveling screw makes it possible to tip the table slightly when desired.

The "molecules" are BB shot, which are sufficiently aspherical and rough that they pick up considerable energy from the motion of the table. They serve much better than do steel balls, which are hardly affected by the motion. One or two $\frac{3}{8}$-in. steel balls among a number of shot serve admirably to demonstrate Brownian motion.

The motion of the particles is shown by projecting their shadows on a screen. The source of light is the anode of a carbon arc. Two mirrors—one below and the other above the shaking table—direct the beam of light as required. Masks below the table can be adjusted so that only the particles can be seen on the screen, the shadow of the table being cut off.

With some 50 shot on the table the conditions are closely approximate to those in a gas. As more shot are added, quasi-crystalling groups appear, as in liquids, though for a different reason, of course. The solid state is represented by a single layer of particles representing a monomolecular thin section through one of the principal planes of a crystal. Beginning with the gas phase on a level table, one may simulate the dynamic equilibrium between liquid and vapor by tilting the table slightly. Change of vapor pressure with temperature can be shown by increasing the amplitude of vibration. A thin paper strip used to confine the particles

to one half of the table will demonstrate vapor pressure. Sublimation can be shown by tilting the table more and adding some 150 particles.

Amorphous solids may be reproduced by nearly filling the level table with particles and running the driving clock in synchronism with the torsional vibration of the table. When the amorphous solid is established, very slight tilting will bring about slow "cooling," eventually resulting in a packing of a few crystals. The fuzziness of the edges of the shadows of individual particles causes an apparent bonding between particles when they are close together.

Osmosis is demonstrated by using particles of two different sizes. The "semipermeable membrane" is a strip of paper across the table, at such a height that the smaller particles can roll under it but the larger ones are stopped.—W. S. Von Arx, *J. Chem. Ed.* **22**, 57–63 (1945).

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